Guidelines for Evaluators

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Appendices*

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* The appendix on Monopole Transition Strength is new. The other appendices have appeared in various documents from NNDC or IAEA. They have been collected together here for the convenience of the evaluators.

1. Source Datasets (Decay and Reaction)

1.1. Extraction of Data

1. In any experiment, the author's basic measured quantities should be quoted as given, except as noted in **Comment 2**, or unless the data can be converted, by the application of known numerical factors, from the author's units to units used by convention in ENSDF. Examples of the latter are the conversion of mean-life to half-life, or BE2(sp) to BE2↑.

2. Give what was actually measured in an experiment and not what the author quotes, in cases where these are different.

Example 1: A measurement of $I_\gamma/\Sigma I_\beta$ might be quoted by an author as I_β(gs), which, for the author's decay scheme should be equivalent to the absolute I_γ determination, but is not as fundamental a quantity. If the decay scheme were to change, the I_β(gs) could change, whereas the absolute I_γ measurement would still be valid. **This distinction is an important one, and failure to make it is a particularly common source of confusion when normalization conditions are being stated.**

Example 2: A measurement of $I_{\gamma\pm}/I_\gamma$ might be quoted by an author as $I_\beta{}^+/I_\gamma$. The ratio should be expressed in terms of the annihilation radiation since $I_{\beta+}/I_\gamma$ implies that the positron spectrum was measured.

Example 3: As discussed in the section on (γ,γ') below, the quantity usually measured is $gW(\theta)\Gamma(\gamma_0)^2/\Gamma$. An author may quote a deduced value of Γ [or $\Gamma(\gamma_0)$] based on an assumption or a measurement of the branching ratio, $\Gamma(\gamma_0)/\Gamma$. If possible, the value of $gW(\theta)\Gamma(\gamma_0)^2/\Gamma$ that the author measured should be "reconstructed" and given, since your adopted branching may differ from what the author used. Note that the author may also have used a different value of J_f in the term $g=(2J_f+1)/(2J_i+1)$.

Example 4: From angular distribution and angular correlation experiments, the resulting γ character can be determined only as dipole, quadrupole, dipole+quadrupole (D, Q, D+Q), etc. Authors sometimes convert these to (M1), (E1), etc., based just on their proposed level scheme. It is important to retain the D, Q etc., assignment in these source datasets. Note that for Jπ assignments, the argument mult=D is strong, whereas mult=(M1) or (E1) is weak. Sometimes mult=D is sufficient as part of an argument, and this is one reason why it is important to keep track of what was actually experimentally determined.

Note: If the half-life of the parent level is known, and, for example, mult=M2 can be excluded by RUL, then the assignment Q=E2 can be made in that dataset. Also, a large value of δ for D+Q may allow M2 to be ruled out, so that the multipolarity can be assigned as M1+E2. In either case the justification should be stated.

3. Document any and all changes made in data quoted from an author. When correcting an author's value for a quantity, for example a misprint in Eγ, give the corrected value in the appropriate field and mention the uncorrected value in a comment. **Do not give the uncorrected value in the field and rely on the comment to explain what the correct value is.**

4. When extracting data from an author's paper, note any assumptions, standards, or constants that enter into derived values, and if possible, correct for more recent values of these quantities. Such corrected values should be explained in such a way that the effect of changes in any of the assumed values is made clear.

> **Example 1:** An I_ε/I_α branching ratio for a ground-state or metastable-state decay for one nuclide might depend on the value assumed for the daughter nuclide; thus, a comment such as "I_ε/I_α=X if I_ε/I_α(α daughter)=Y" would be appropriate.
>
> **Example 2:** In a decay dataset, a conversion coefficient might be given relative to a standard value. A comment such as "α_K=A if α_K(standard)=B" would be appropriate.

5. Check the bibliography in each article against the NSR file. This is a valuable cross check to help ensure that references have not been overlooked. Also, an author will sometimes quote data received as a private communication. These data should be tracked down if possible if they are important. Any new references obtained in this manner should be communicated to the manager of the NSR file.

6. Do not rely on an author to extract older data correctly. Even if an author collects such data in a table, the original articles should be checked. This is especially important in view of point **3**. above.

7. Be sure to distinguish between values measured by an author and those assumed by the author based on data from other work.

> **Example 1**: In a transfer reaction, an author might adopt L values for some levels based on known Jπ values or on L values determined by other authors in order to determine standard shapes for σ(θ) as a function of L so that L values for other levels can be deduced. These "assumed" L values should be given in square brackets.
>
> **Example 2:** In determining E_γ or E_{level}, an author might adopt values from other work as internal calibration standards. These standards should be checked and noted. If newer values for the calibration standards are available, it might be possible to correct the authors' energies. The quality of the data will dictate whether or not that is worth doing.

1.2. Manipulation and Presentation of Data

1.2.1. Comments

See the ENSDF manual for a complete discussion of the formats for Comments.

1. **Comment types:** There are several types of comment formats differing in where they appear within the headings and data listings. All require a "C ", "T ", or "D" in column 7.

   a) **No entry in column 8 or 9**. These comments apply to the dataset as a whole and appear at the head of a dataset immediately following the ID record.

   b) **A record type (RTYPE) such as "L", "G", or "B" in column 8, with no data type modifier (SYM) in column 10.** These comments appear at the head of the respective level, gamma, or beta listing.

   c) **Same as b). but with a data type (SYM) modifier in column 10, such as "E", "RI", or "CC".** These modifiers should be followed by the delimiter "$" to separate the modifier from the comment text, thus "E$" etc. If entered at the head of a dataset, these comments appears as footnotes on the respective column headings in the data listing defined by the content of column 8. If entered within a data listing, the comment appears in the Comments column on the same line as the quantity being commented on.

   d) **Same as b). but with a flagged modifier in column 10, such as "E(A)", "RI(B)", "C(D)" or "E(E),RI(E)" and entered at the head of a dataset**. As above, a delimiter is needed. By using a "FLAG=A" continuation record on a specific data entry within the data listing, a footnote symbol is attached to that entry and the corresponding comment appears as a footnote. An alternative is to put the flag symbol in column 77 of the data entry.

      Note: The use of the flagged comment format is recommended whenever the same comment applies to many data entries, especially if the comment is long and involved. This avoids having the Comments column filled with many appearances of the same comment

2. In all datasets there should be comments of type **1. a)** for each keynumber that contains data or information used in that dataset. In cases where the same data are given in more than one publication, avoid giving a separate entry for each such reference. Choose the most complete reference, and on the same line, state something like "These data are also reported in xxxxx". This policy tells the reader which references should be looked at and which can be neglected.

   Note 1: References listed as "Others:" do not need detailed information.

   Note 2.: In listing the keynumbers, for ease in readability, it is recommended that each be given on a separate line rather than running them together across the page.

a) For decay datasets, the comments for each keynumber should give the type of data measured, and there should also be a comment stating which references have contributed to the decay scheme being adopted.

> **Note:** Including what was deduced is optional. The deduced quantities are often obvious by looking at what was measured, but there might be special cases where such information is useful.

Example 1: 1992Ma42: Measured E_γ, I_γ, $\gamma\gamma$, $\beta\gamma(t)$. Also reported in 1993Ma42
1975Wi08: Measured E_γ, I_γ, $E_{\beta-}$, $I_{\beta-}$, $\gamma\gamma$, $\beta\gamma$

Example 2a: For the case of a simple decay scheme the following is sufficient.
"The decay scheme is that proposed by 1975Ko02 based on extensive $\gamma\gamma$ data".

Example 2b: In a more complicated case the following comment was written.
"The decay scheme is that proposed by 1983Dz01 based on extensive coincidence data using the E_γ and I_γ data of 1971Va26. The 1263 level was added by the evaluator on the basis of the agreement in energy and branching of the 576 and 1263 γ's with those reported in (p,2nγ). The 1420 level is proposed by 1985Ra21 and is also reported in (p,2nγ)"

> **Note:** The decay scheme should be traceable to its source, so as much detail as is needed to accomplish this should be included.

b) For reaction datasets, the comments for each keynumber should include the bombarding energy and energy resolution. If the energy resolution is not given explicitly by an author, the evaluator should attempt to estimate it from the authors' spectrum.

> **Note 1:** The resolution is an important consideration when comparing the results from two or more experiments of the same type, or when making level associations in Adopted Levels.
>
> **Note 2:** Do not put the bombarding energy on the ID record unless needed to distinguish otherwise identical dataset ID's such as E=th and E=res for (n,γ). The bombarding energy should be part of the information given in comments for each keynumber.

Other information such as angular range for $\sigma(\theta)$ measurements might be included since, for example, small angles are often needed to establish L=0 transfers and the angular range studied might lead to a preference of one experiment over another in cases of discrepancies. Except for even-even targets, Jπ(target)* should be given.

* For radioactive beam experiments, Jπ of the beam is the quantity needed.

For grouped reactions, such as (HI,xnγ) in **Example 3** below, the specific reaction should be given, and for Coulomb Excitation in **Example 2** below, the distinction between particle detection, (x,x') and gamma detection, (x,x'γ) should be made. For (γ,γ') datasets as in **Example 4** below, the source and the excitation Eγ are required.

Example 1: ^{207}Pb(d,p),(pol d,p)
Jπ(target)=1/2^{-}
2001Va04: E(d), E(pol d)=22 MeV, FWHM=5-6 keV
2006He21: E=22 MeV, FWHM=3 keV
Others: 1968Do04, 1967Ba41, 1962Mu05

Example 2: Coulomb Excitation
1969Ba51: (x,x') x=α, E=17-19 MeV; x=^{16}O, E=69 MeV
1972Ha59: (x,x'γ) x=α, E=15, 18 MeV
1959Bi10 : (x,x'γ) x=p, E=2.8 MeV, pulsed beam
1971Di02: (x,x'γ) x=^{40}Ar, E not given, recoil distance

Example 3: (HI,xnγ)
1983Ba70: ^{130}Te(^{27}Al,5nγ), E=154 MeV
1995Pe16: ^{146}Nd(^{11}B,5nγ), E=66 MeV

Example 4: (γ,γ')
1963Fl04: source = ^{56}Fe(n,γ), $E_γ$=7279
1973Sw01: source = Doppler-broadened 7117γ from ^{19}F(p,αγ)
1977Co10: source = bremsstrahlung, E(max) = 6600, 9700

c) Comments on γγ(θ), γ(t), γ(θ), γ(θ,H,T) etc., in a given dataset should normally be given with the listing for which the information is relevant. γ(t) and γ(θ,H,T) would normally appear with the levels listing since the information usually derived from experiments of these types, T½ and g-factors, respectively, are properties of the level. Comments on γγ(θ) and γ(θ), including the distribution coefficients, A_2, A_4, or DCO ratios etc., from which the multipolarity and δ might be derived, should be given with the transitions involved. If information on J is also deduced, that J value should be given with the level listing, with a cross reference to the γ listing, for example "J=3/2 from γγ(θ). See γ listing for details".

d) Other required comments

i) For the β intensities in the β listing there should be a comment or footnote stating how the values were obtained. For example, something like "Unless noted otherwise, the I(β's) are from the requirement of an I(γ+ce) balance at each level" would be appropriate.

ii) For the gamma listing, there should be a comment stating how the normalization was obtained, and a footnote stating what the entries are, that is "relative photon intensity", "% branching from each level" etc.

iii) For the levels listing, there should be a comment on the E_{level} heading giving the source. The following is one example. "For levels with γ's, the excitation energies are from a least-squares fit to the adopted Eγ data. For other bound levels the energies are weighted averages from all reactions. For proton resonances the excitation energies have been calculated from E(p) using S(p)=xxxxx"

Note: The E(level) heading comment could also include mention of levels that are not adopted, for example broad peaks, or could include any other general points the evaluator wishes to make. For example, "In addition to the levels listed, broad peaks are reported in (d,n) at 2540 *80*, and 3720 *120*. Above about 2500, the association of levels seen in the different reactions is uncertain. The evaluator has chosen to show separate levels in such cases; however, it is possible that where the energies overlap these reactions may be populating the same level".

1.2.2. Combining Datasets

It is sometimes convenient to combine two or more reactions in a single dataset. This approach is useful when the reactions are similar in nature and where data in the reactions are sparse, or where one reaction is very complete with little information in the others. The following are two situations where this might be appropriate.

1. Inelastic scattering experiments, (p,p'), (d,d'), (α,α') etc., can sometimes be combined into a single dataset as (x,x'). If this is done, then of course it is important to specify in which reaction a property such as L or S was determined. The following example compresses 10 separate datasets into one . See the 2013 version of the A=152 mass chain for an example of the data presentation. The following is an excerpt.

^{152}Sm(x,x')

x=n:
1985Fe04: E=2.47, 2.75 MeV, σ(θ); coupled channel analysis; levels 122, 366, 1086
x=e:
1988Ph01: E=251, 500 MeV, σ(E,θ): levels 122, 366, 707

x=p, pol p:
1993Pe01: E(pol p)=20.4 MeV, σ(θ), analyzing power; levels 0, 122, 366, 707, 963, 1041, 1221
1989Ob02: E=24 MeV, FWHM=18 keV, σ(θ); levels 0, 122, 366, 707

etc.

Note: The notation for inelastic scattering, (x,x'), includes the case of elastic scattering, so there is usually no need for a separate (x,x) dataset. One exception is resonance work, where information on resonances in the compound nucleus can be obtained and may be of importance (see **section 1.5. Resonances, page 22)**. Information on nuclear shapes, charge densities, etc., deduced from elastic scattering can be given, or referred to, in Adopted Levels without the need for an (x,x) source data set.

2. In a similar fashion to inelastic scattering in **1.**, heavy ion in-beam reactions can sometimes be combined into a single dataset as (HI,xnγ). Since I_γ data from different in-beam reactions can be combined only via branching ratios, if there are several sets of fairly complete relative intensities, it is best to create separate datasets for those reactions; however, note that the TI column can be re-labeled as I_γ, allowing for the inclusion of two sets of relative I_γ values or of branching ratios, or one of each, within a single dataset.

1.2.3. Sources of data

The sources of data for all column headings, E(level), Iγ, δ, L, etc., should be given in footnotes on the corresponding heading, unless there is only one reference for that dataset. In this case, the reference can be given with the ID record, and/or a comment can be given stating "All data are from xxx". Keep in mind that all the data presented should be readily traceable to their source. When more than one keynumber is included on an ID record or in the heading comments, it is important to state from which keynumber the individual pieces of data are taken. If a reader wants to check an E_γ, an I_γ, or a δ, for example, that reader should be able to go directly to the relevant reference, or references. If the E(level) values come from a least-squares adjustment to the E_γ values, this should be stated explicitly.

1.2.4. Placement of gamma records

For consistency in presenting drawings (and for convenience in reading data base listings) gammas should be placed in order of increasing energy following each level. This same order should be followed in listing the unplaced gammas. Note that Format check will warn you if gammas are out of order.

1.2.5. Significant digits

When converting values from one set of units to another, for example, half-life to mean-life, or when re-normalizing Iγ values, enough digits should be retained so that the inverse operation will reproduce the original values. Note that in some cases this will result in more digits being quoted in the converted value than in the original value. Another way of stating this principle is that the fractional uncertainty in the original value should be preserved (to the same number of significant digits) in the converted value. This procedure is especially important when dealing with quantities determined with fairly high precision.

Example 1: From BE2=0.384 *4* one should report T½=7.27 ps *8*, not 7.3 ps *1*, and from τ=32 ps 1, one should report T½=22.2 ps *7*, not 22 ps *1*.

Example 2: Note that except for Adopted Gammas datasets it is not necessary to re-normalize data to Iγ=100 for the strongest transition. Some re-normalization may be useful when averaging two or more sets of values, but in a case where there is just one set of values and the authors do not assign Iγ =100 to any of the transitions, there is no need to re-normalize the data. Consider a case where the author has assigned the strongest transition as I_γ=90 *3*. A re-normalization to I_γ=100 gives 100.0 *33*, and unless the large uncertainty is kept, this creates a problem with the roundoff for this transition and probably for many of the others. If these data are then used in Adopted Gammas, there will be an additional re-normalization involved in getting branching ratios and a probable further change from the original fractional uncertainties. It is recommended to keep re-normalizations to a minimum.

Note: As a special case of **Example 2**. if an author gives absolute intensities, these should definitely not be re-normalized in a source dataset.

1.2.6. Data roundoff

1. Our present suggested upper limit for rounding off uncertainties is "25". There are cases where this cutoff should be increased.

Example 1: When two or more values are being averaged, and the uncertainties are comparable, with some just below the cutoff of "25" and some just above, then it is recommended to take the average before rounding off the values with uncertainties >25.

Example 2: When the fractional uncertainty is large, retaining uncertainties >*25* may be justified. A value of 5.8 *27* for a quantity should be kept, rather than rounding off to 6 *3*.

Example 3: When the uncertainty on a value gives a lower limit close to zero, a roundoff may be misleading. A value for some quantity of 3.2 *26* does not overlap zero, whereas a rounded-off value of 3 *3* allows for the quantity to be zero.

2. Do not replace numerical values with large uncertainties by approximate values.

Example: An "isomer" energy of 230 *300* allows for the possibility that the isomer may lie below the "ground state" by 70 keV. If the energy is replaced by ≈230, the possibility of an isomer-ground state energy inversion will not be obvious to the reader.

1.2.7. Multiplets

1. In a reaction spectrum, unless a peak labeled as "complex" is resolved in a given experiment, just one "level" entry should be made. In the case of a peak suspected on the basis of work from other experiments of being made up of two levels, with known Jπ, say J =a and J =b, respectively, if a value for Jπ for the single "level" entry is to be entered, it should be of the form "J =a and b", or "J=a & b". The inclusion in this data set of two levels would involve making an explicit assumption that is not necessary. The probable level association can be adequately explained in a comment.

> **Note 1:** The form “J=a and b” denotes a multiplet with two Jπ components. The form “J=a, b” implies one level with two possible Jπ values.
>
> **Note 2:** When stating that a peak is a multiplet, the basis for this claim should be given. In this connection it is important to distinguish between experimental arguments such as "peak is broad", and theoretical arguments such as "C^2S is too large for a single level on the basis of shell model expectations".

2. In a gamma spectrum, a multiply placed transition seen as a single peak in the spectrum should be treated as one transition with multiple placements. Do not "resolve" such peaks by introducing additional transitions with energies taken from the level scheme. Of course the intensity should be divided among the several placements if possible. If the intensity can be divided, for example on the basis of γγ, or from branching ratios in other datasets, then a "@" should be entered in column 77. Comments such as "From γγ" or "From $I\gamma(\gamma_1)/I\gamma(\gamma_2)$ in β^- decay" should be given. If the intensity cannot be divided among the several placements, then the full intensity, with uncertainty, should be given for each placement, along with a "&" in column 77. The entries "&" and "@" will automatically generate footnotes explaining that the transitions are multiply placed and that the intensities are not divided (for "&"), or are suitably divided (for "@").

> **Note 1:** Do not enter the intensities as limits in source data sets. The converse is true in Adopted Gammas, where multiply-placed Iγ values should be entered as upper limits. See **section 3.3. Iγ, Comment** 2, **Note 2, page 43.**
>
> **Note 2:** When stating that a gamma transition is a multiplet, the basis for this claim should be given. For example, the gamma peak might be broad, coincidence data might suggest that a peak is a multiplet or a comparison of Iγ branching with other transitions from the same level as determined in other datasets might suggest a multiplet.

3. A multipolarity determined for a multiplet will not necessarily be correct for each, or perhaps even any, member of the multiplet. For example, depending on the relative strengths of the components, the I_γ and I_{cek} for a doublet consisting of an El and Ml component could yield mult=E2. The multipolarity deduced for the doublet should be given in a comment, but should not be entered in the multipolarity field of the individual components unless additional information is available that justifies the assignment.

> **Note**: In a case where I_γ but not the corresponding I_{cek} (or vice-versa) is resolved, and the multipolarity of one component of a doublet is known from other sources, it may

be possible to deduce the multipolarity for the other component.

Example: The authors report Eγ=1411.48 *9* with Iγ=1.01 *3* placed from levels A and B, with Iγ divided between the two placements based on γγ. Mult=M1+E2 with δ=+4.3 *+9-13* for the component from level B is known from other work. From these data and α(K)exp=0.00083 *14* for the doublet, mult=E1 is deduced for the component from level A.

1.2.8. (γ,γ') experiments

The most common type of measurement in these experiments is scattering, which, for the case of photons scattered elastically from a thin target, yields the quantity $gW(\theta)\Gamma(\gamma_0)^2/\Gamma$. This is what most authors quote. Here, $g=(2J+1)/(2J_0+1)$, with J=γ-resonance level spin, J_0=gs spin, and W is the usual angular correlation function (See **Note 1**). In this type of experiment the quantity $gW\Gamma(\gamma_0)^2/\Gamma$, or just $\Gamma(\gamma_0)^2/\Gamma$, if J and W are known, should be given. If the branching $\Gamma(\gamma_0)/\Gamma=I_\gamma(\gamma_0)/\Sigma I_{\gamma+ce}$ is known, the level width (or T½) should be deduced. For levels above particle decay separation energies, then the total width should include the additional modes of decay. That is, is general, $\Gamma=\Gamma_\gamma+\Gamma_p+\Gamma_\alpha+\ldots$ The branching used should be the adopted value.

Note 1: Measurements are usually done at 127° where W=1 for all dipole transitions, independent of J_0, J, or J_i (see **Note 5**). ($P_2(\theta)=0$ at this angle). For mixed transitions, W depends on the mixing ratio and on the J's.

Note 2: Occasionally, self-absorption experiments are performed. These can yield the quantity $gW\Gamma(\gamma_0)/\Gamma$.

Note 3: The quantity $gW\Gamma(\gamma_0)^2/\Gamma$, with g and W taken out if known (at the evaluator's discretion), can be given in the "S" field on the level record, with the field suitably re-labeled. This procedure is convenient since it eliminates considerable typing work at the input stage. If given in units of milli-electron volts, be sure that the heading translates as "meV" and not "MeV".

Note 4: If the branching, $\Gamma(\gamma_0)/\Gamma$, is measured, it can be given in the RI field for the relevant γ or as a comment on branching given with the corresponding level.

Note 5: For inelastic scattering, the term $(\Gamma_0)^2$ in the numerator should be replaced by $\Gamma(\gamma_0)\Gamma(\gamma_i)$, where (γ_i) refers to the deexciting transition to excited level "i". To extract the level width (or T½), in addition to the gs branching, the ratio $I_\gamma(\gamma_i)/I_\gamma(\gamma_0)$, would be needed.

1.2.9. BEλ, BMλ, and βλ

In Coulomb excitation and (e,e'), two reactions where electromagnetic excitation probabilities can be determined, the quantities BE2, BE3, etc., should be quoted on continuation level records. Data quoted as matrix elements should be converted to BE2 etc. The fact that a matrix element had been determined should be added as a comment. Note that $BE\lambda = (2J_0+I)^{-1}\,|<ME\lambda>|^2$, where $<ME\lambda>$ is the Eλ matrix element and J_0 is the target spin.

> **Note 1:** All BEλ and BMλ data should be given with the levels. If an author gives BEλ↓ data with the gammas, these should be converted to BEλ↑ and given with the corresponding level. The appropriate place for BEλ↓ or BMλ↓ data is in Adopted Gammas where such values are given in single-particle units calculated using adopted T½, δ, and branching, etc., data.
>
> **Note 2:** It is not necessary to give T½ deduced from BE2 in the source dataset, but if done, then adopted values for Eγ, branching, etc., should be used. It is often more convenient to collect all T½ values in Adopted Levels in which case one can state "From BE2=xxx in Coulomb Excitation" for the value from that dataset.

In inelastic reactions other than those governed by the electromagnetic interaction, the appropriate interaction strengths to quote are the deformation parameters, βλ or βλR, where R is the nuclear radius, $R_0A^{1/3}$. Authors sometimes convert the deformation parameters to BEλ's, but this is a model-dependent procedure and unless the authors quote only BEλ's, the deformation parameters are what should be entered from these experiments.

1.2.10. Isomeric decay and delayed gammas

When a level with a measurable half-life has been produced and studied as a separate source, then an isomeric decay dataset for that parent level should be created. When delayed gammas are seen in an in-beam experiment, creating an isomeric decay dataset is sometimes a convenient way of presenting the delayed data, as discussed in **2.** below.

1. When in-beam experiments give data on prompt transitions and also on delayed transitions from, say level X, one way of presenting the data is to create two datasets, one labeled with the modifier "prompt gammas" and the other with the modifier "delayed gammas" where the data for decay of level X can be presented. For cases where there is more than one level whose delayed deexcitation has been observed, a separate "delayed gammas" dataset can be prepared for each such level. See the following comment.

2. An alternate mode of presentation for delayed gammas is to create an isomeric decay dataset for level X . This alternative is especially recommended if there is more than one reaction yielding data. In this case a single IT data set which combines the results from all the relevant reactions is preferable to creating several delayed-gammas data sets for the same level X in each of the several reactions.

> **Note:** In cases where only a few pieces of data come from the delayed spectrum, such as multipolarities, it may not be worthwhile creating a separate delayed or isomeric decay dataset. In such cases, the multipolarity information can be included with the prompt data with appropriate comments. For example. "Mult: From α=xxx from an intensity balance in a delayed spectrum".

1.2.11. β^-, $\varepsilon+\beta^+$: energies, feedings and logft

1. Measurements of E_β or E_ε should be given in comments rather than in the energy field. All of our programs deduce these quantities from the Q value and the level energies.

Note: If a new accurate measurement of E_β or E_ε appears in the literature and that value has not yet been included in the latest mass adjustment, then it should be compared with the input values in that adjustment. If it is of comparable accuracy, an attempt should be made to see if it alters significantly the output value from the mass adjustment. If it does, an updated Q value can be adopted. In this regard, correspondence with the atomic mass center is advised.

The β^- and $\varepsilon+\beta^+$ feedings usually come from intensity balances at each level, and the logft's are then calculated based on these feedings. Some special cases should be noted.

2. For transitions that are expected to be non-negligible, such as those for allowed or first-forbidden transitions, when the feedings are consistent with zero, the logft should be expressed as a lower limit.

Example 1: For a feeding of 3% *3*, the logft should be calculated for a feeding of 6% and expressed as a lower limit.

Example 2: For a feeding of -5% *7*, the logft should be calculated for a feeding of 2% and expressed as a lower limit.

3. When $\Delta J>2$, one expects logft$\gtrsim$13. For such transitions, any feeding that gives logft$\lesssim$13 should be set to zero, with an appropriate comment, since such feedings imply an error in the decay scheme. An exception to this policy of omitting "unphysical" branches occurs when the initial or final J is in question and it is not clear whether it is the spin or the feeding that is in error. In such a case, the feeding should be shown with a "?", and the problem pointed out in a comment.

4. The combined feeding to two or more levels connected by γ transitions whose TI's are not known, or known only as limits, can sometimes be determined even though the individual feedings cannot. Such combined feedings should be given in a comment.

Example: "I_β- to the X and/or Y levels is 6.2% *2* giving logft=xxx". This could be given as a flagged footnote for each level.

1.2.12. Normalization

The normalization condition giving NR, NT, should always be stated. If the decay scheme cannot be normalized, this fact should be pointed out. Be sure to account for all relevant factors.

Note 1: If the normalization condition involves a measured quantity for which no uncertainty is quoted by the authors, for example, I_{β}-(gs)=30%, then the resulting NR should be given as approximate. If NR is given with no uncertainty, GTOL will generate level feedings, and RadList will generate absolute intensities that reflect only the uncertainties in the relative intensities. In the example given, if the evaluator chooses to assign an uncertainty, $\Delta I_{\beta -}$, that uncertainty can be explicitly added to the I_{β}- in the listing, with an appropriate comment, or simply referred to in the normalization statement, for example "NR: The evaluator has assigned an uncertainty of x% to the intensity of the gs β^- branch in order to get an overall uncertainty for NR". The second approach is recommended since it preserves the accuracy of the relative feedings. Note that when the gs branch has a small intensity, say a few percent, then even a large assigned uncertainty can result in a rather precise NR as calculated from TI(gs)=100-I_{β}-(gs). See also **section 1.4. Uncertainties, Comment 2, Note 1 page 19**.

Note 2: In a case where the I_{γ}'s in the RI field already include all the uncertainty appropriate for absolute intensities, such as when an author determines and quotes absolute values (including absolute uncertainties), then no additional uncertainties should be given with the normalization factors. That is, NR, NT, and BR should be given on the "N" record with no uncertainty. There is no requirement that the uncertainty in BR given in Adopted Levels be carried over to the "N" record in a decay data set, although the value itself of course must be the same.

1.2.13. Parent records

In the parent record, the fields where data are known should be filled in, and the data should be the same as in the adopted dataset for the parent, unless the evaluator is updating those data. Comments on "P" record data, such as sources, are not needed unless the evaluator is modifying them. The appropriate place for comments on any of the quantities appearing on the "P" record is in the adopted data set for the parent nuclide.

1.2.14. α Decay

1. If the energies of the daughter levels being fed are not known, the E_{level}=0+x style should be used rather than listing the alphas as unplaced. With this procedure, relative level energies can be presented in the daughter mass chain. Note that there is no such thing as an unplaced alpha, except an alpha whose parent assignment is uncertain.

2. The calibration standards used by an author should be checked for possible newer values and the E_{α}'s corrected accordingly. The evaluation by Rytz, 1991Ry01, is still a useful starting point for checking calibration standards but recent mass chains should also be consulted for possible newer precise values. In this regard, see the comments by D. Abriola, section 3.4 of the IAEA report *Summary Report of a Specialised Workshop on Nuclear Structure and Decay Data (NSDD) Evaluations*, **INDC(NDS)-0688**, April, 2015.

3. Hindrance factors should be given where possible. See **Appendix A** for a description of how to get the radius parameters needed for the hindrance factor calculations.

1.2.15. Half-lives*

For decay datasets, the adopted T½ should be entered for all levels. Values measured in a source dataset should be given in a comment along with the method and the keynumber. For reaction datasets, measured values should be given in the T½ field.

* See also the footnote on **T½**, **section 2.4**, page 42.

Note: Footnotes on the T½ column can be used to cut down on repetition of the method/keynumber requirement. For example, a footnote stating "T½ values given as comments are from γγ(t) (keynumber) except where noted otherwise", or "From adopted levels except where noted otherwise", would be appropriate.

1.2.16. Miscellaneous

1. The symbol "/" should not be used when proportionality of more than two values is being expressed. The expression K/L/M is mathematically equivalent to KM/L. Use ":" instead, thus K:L:M.

2. Try to resolve discrepancies. If they cannot be resolved, then at least state this fact.

> **Example:** If δ=+0.38 *4* is adopted for a certain transition, based on solid evidence, and a value δ=+1.2 *3* has been determined in one of the source data sets, then, if the reason for the difference cannot be determined, the evaluator should at least point out the discrepancy. This can be done with a comment in the source data set, where it could state that the value differs from the adopted value, or with a comment in adopted γ's, where the discrepant value could be mentioned. If the discrepancy is not pointed out, the reader might think that the discrepant value had been overlooked and might thus question the adopted value.

3. Use the word "uncertainty" rather than "error" to refer to what we call the standard deviation in a measured quantity. The word "error" should be reserved for mistakes, such as typos, or inconsistencies in values for the same quantity within a paper.

4. Note that TI is translated in the ENSDF output as I(γ +ce), not I($\varepsilon+\beta^+$) even though the fields have the same name. When I($\varepsilon+\beta^+$) is what is meant, it must be spelled out.

5. A level designated as an isomer in one data set should be treated as an isomer in all data sets, that is, columns 78 and/or 79 should be filled in.

6. It is not necessary to comment on correction factors for a quantity when such factors are negligible relative to the uncertainty quoted for the quantity. For example, μ=+3.8 *5* does not require a comment stating "The diamagnetic correction has not been applied".

7. Do not use "CA" in the uncertainty field when a numeric uncertainty can be calculated.

Example: If I_γ is calculated from $I_{\gamma\ +ce}$ and α, the uncertainty in I_γ from the uncertainties in TI and α, not the symbol "CA", should be put in the uncertainty field. A comment is of course needed to explain the source of the value.

8. When calculating or correcting quantities that depend on other properties, for example calculating a conversion coefficient which depends on E_γ, multipolarity, and δ, or calculating T½ from BE2 which depends on E_γ, branching, δ, and α, or calculating Γ from $\Gamma(\gamma_0)^2/\Gamma$ which depends on J and the gs γ branching ratio, adopted values should be used for all relevant quantities.

Note 1: When α values are given in any dataset, they should be adopted values, that is, calculated using the adopted E_γ and δ values. A comment on the heading of the α column should be given stating this fact. Alternately, this policy could be stated in a general statement in the Comments dataset at the head of the mass chain.

Note 2: In a dataset in which BE2 has been measured, if T½ is deduced from that BE2 value it should be done using the adopted values for E_γ, γ branching, δ, and α. Note, however, as mentioned in **section 1.2.9. BEλ, BMλ, and βλ, Note 2, page 10**, that it is not always necessary to convert BE2 to T½. in the source dataset.

9. When working with data in any dataset, a comparison with data in other datasets should be made.

Note 1: A γ might be multiply placed in dataset A but resolved in dataset B. The branchings from B can sometimes be used to divide intensities in A. This should be done whenever possible.

Note 2: It is recommended that E_γ and E_{level} data in each dataset be checked against values for the same quantity obtained in other datasets. This cross checking will sometimes show that data in one dataset are shifted relative to those in other datasets. In such a case a comment should be made pointing this out, and the shift should be taken into account in making level associations in Adopted Levels, and in arriving at adopted energies.

10. Measurements of $P_k\omega_k = I_{K\ x\text{-}ray}$ should be given. They can be entered on a continuation "E" record. These quantities are of direct interest to some researchers and of course they provide a direct measurement of the K x-rays, either for branches to individual levels, or an average for the whole decay scheme, depending on the case. When possible, the $P_k\omega_k$ should be compared with $I_{K\ x\text{-}ray}$ as calculated by RadList.

11. If numerical data are quoted in comments, the uncertainty should be included, unless the value is being used only as a label, thus "T½: From BE2=0.240 *6*", or "μ : From g=1.62 *3* in (α,2nγ)". This is not to imply that the actual numerical value is needed in all cross references, but only that if quoted, the uncertainty should be included.

12. When changing the sign of a mixing ratio which has an asymmetric uncertainty, note that $\delta = A$ +a-b becomes $\delta = -A$ +b-a.

13. The ground state should be included in all data sets of the type (X,X') or (X,X'γ).

14. It is recommended that cross sections, analyzing-power values, and angular- distribution coefficients be given explicitly. They can be of importance in justifying any conclusions based on such data. The conclusions themselves, of course, should be given.

> **Note:** The angular-distribution coefficients should be given in the form A_2, A_4, not A_2/A_0, A_4/A_0. That is, we define the angular distribution function as $W(\theta)=1+A_2P_2(\cos\theta)+$, not as $A_0+A_2P_2(\cos\theta)+$.

15. Separate data sets for experiments with no specific level data but with some useful general information can be included at the evaluator's discretion. Such data sets would consist only of comments. The following are examples:

^{208}Pb(e,F)

1976Dr01, 1977Ke11 E=28-44 MeV
1976Tu03 E=38-50 MeV
1976Dr01 and 1977Ke11 determine level density parameters and give a fission barrier of 27.6 MeV *5*. 1976Tu03 determine the fission barrier to be 23.6 MeV *15*

^{154}Sm(α,^{6}He) 1974BoZF,1974BoZN

E=50 MeV. Measured $\sigma(E(^6He),\theta)$

^{67}Zn(n,α)

For studies of the ^{67}Zn(n,α) reaction and the parameters of the the resonances in the compound nuclide ^{68}Zn, see 1978An01, 1984Em01, and 1985Gl04.

16. In some cases the information measured in a reaction could be included directly in Adopted Levels without the need for a separate dataset, for example where T½ for a single excited state was determined. Keep in mind, however, that unless a dataset is created for such a reaction, there is no convenient way to search and retrieve that reaction and thus to indicate to the reader that such a reaction was studied. If that reaction seems important then it should have its own dataset. If a reaction was studied but no "useful" information is available, then it can be simply listed under "Other reactions" as a comment in Adopted Levels.

1.3. Systematics

Use should be made of systematics whenever possible, the extent to which they can be applied in any given case being determined by how reliable they are for the mass region under consideration. The evaluator is usually in a better position to know how and when to apply systematics of a given quantity than the typical reader who is generally looking at just one, or perhaps a few, nuclides at a time.

> **Note:** The network evaluators already make extensive use of systematics. The strong arguments for Jπ assignments which rely on logft's, the strong arguments for multipolarities which rely on RUL, and extrapolations from the measured data in the mass adjustment (which are in fact called systematics values) are perhaps the prime examples.

1. The gross beta decay T½(β^-) and T½($\varepsilon+\beta^+$) estimates from K. Takahashi, et al., Atomic Data and Nuclear Data Tables **12**, 101 (1973) (1973Ta30), can be used to estimate β^- or $\varepsilon+\beta^+$ branching fractions. These half-life estimates have been found to be reliable to better than a factor of about 3 for nuclides not too far from stability; thus, while an estimate of % β^- ≈50, and thus branching for the alternate modes ≈50%, is perhaps of marginal usefulness, an estimate of % β^- ≈0.1 can be used to assign the alternate mode(s) as essentially 100% with a high degree of confidence. Another source is P. Moller et al., Atomic Data and Nuclear Data Tables **66**, 131 (1997) (1997Mo25).

> **Example:** ^{106}Te has been observed to decay by a single α group to the gs of ^{102}Sn. T½=70 us *17* for this α branch. Decay via an $\varepsilon+\beta^+$ branch is also allowed but has not been observed. From the graphs in 1973Ta30 one gets T½($\varepsilon+\beta^+$)>0.1 s which leads to %($\varepsilon+\beta^+$)<0.09. It is thus reasonable to adopt %α=100.

2. Systematics of alpha-decay hindrance factors can be used to deduce a variety of quantities, depending on what is known about the decay branch. These quantities include Jπ and configurations, total alpha branching and branchings of individual groups, and the excitation energy of the level fed in the daughter nucleus. Each evaluator (or center) responsible for a mass region in which alpha decay occurs is encouraged to build up such a set of systematics. See M. R. Schmorak, Nuclear Data Sheets **31**, 283 (1980), and M. R. Schmorak, Alpha-Decay Hindrance Factors, in **Appendix E** for a further discussion of these and other types of systematics.

> **Example 1:** For α-decay of an even-even parent, the gs branch is defined as having a hindrance factor HF=1, Other hindrance factors are defined relative to this value. As pointed out in **Appendix E**, in the mass region A>214, the HF's for 0^+ even-even parents to the 2^+ first excited daughter states vary smoothly as a function of A from 0.9 to 4.0. For the case discussed in the **Example** in **1.** above, given E(2+)=1472 for ^{102}Sn, one gets %α<2×10^{-6} for a possible α branch to this state. It is thus safe to assign I(α)=100% to the observed α group.

Example 2: Prior to 2002, for ^{110}Xe α decay, only Eα had been measured. From systematics of r_0 values, a partial T½ for α decay of ^{110}Xe is estimated to be T½(α)=0.06 s *+10-3*. From 1973Ta30 (see **Comment 1.** above) one estimates T½(β^+)=0.5 s +5-3. From these partial T½ values one gets %α=87 *+10-32* and %(ε+β^+)=13 *+32-10* and thus T½(^{110}Xe)=54 ms *+84-28* (see 1998Ak04). Measured values for these quantities for ^{110}Xe from 2002 reported in the 2012 version of the Nuclear Data Sheets are T½=93 ms *3* and %α=64 *35*. These measurements indicate the usefulness of the interim values adopted based on systematics.

Example 3: For ^{172}Os, measured values for %α were, prior to 2004, 1.0 *2* and 0.2 with no uncertainty. The smaller value gave a radius parameter r_0 inconsistent with the value expected from systematics and consequently the larger value was adopted. In 2004 %α was re-measured as 1.4 *3*, confirming the interim choice adopted on the basis of r_0 systematics.

3. In cases where a certain shell-model state or Nilsson-model orbital, or a pair of such states or orbitals gives rise to the appearance of isomeric states over a reasonably large mass range, the reduced transition probabilities for the isomeric transitions usually fall within a narrow range of values. Such values can be used to estimate properties for the "same" transition in nuclides where one piece of information, such as T½, IT branching, or E_γ, is missing.

Example: In the region around mass A=95 there are many cases of $1/2^-$ to $9/2^+$ isomeric M4 transitions. In particular, for Nb, the BM4W values vary smoothly from 18.6 *3* for ^{91}Nb to 6.03 *21* for ^{97}Nb. The isomeric branch is not known for ^{99}Nb, but an extrapolation suggests a BM4W of 4.0 *10*. From this systematics value one extracts %IT=0.32 *8* In the A=99 mass chain evaluation %IT was adopted as <3.8% based on the RUL limit of 30. The systematics of BM4W values in this region suggest a smaller non-zero limit.

4. In cases where a ground-state β^- or ε branch is not known but the initial and final Jπ for the branch are known, and there is no other way to determine the intensity normalization for the γ's, the systematic logft values can sometimes be invoked. Moreover, it might be possible to build up local systematics of logft values for similar transitions, that is, transitions involving the same configurations, that give a more restrictive range of logft values.

Example: For the ε+β^+ decay of ^{152}Ho, the intensity of the gs transition has not been measured. The transition is 2^- to 0^+, which from logf^{1u}t>8.5 gives I(ε+β^+)<15%, and thus $\Sigma I_{\gamma+ce}$>85%, or 92% *8* for calculational purposes; however, there are three similar transitions in nearby nuclides with logf^{1u}t values ranging from 8.8 to 9.9. If one uses a lower limit of 8.8, one gets an expected gs branch of <7%. which gives a normalization condition ΣI(γ+ce)>93%, or 96% *4*. The representation of the limit as a value with an uncertainty is done here just for the purpose of getting a value for the normalization. A normalization given as a limit is of course essentially useless.

1.4. Uncertainties

1. Experimental uncertainties should be included whenever given by an author.

> **Note:** If an author gives a general statement, you should include this and then also explain how you interpret that statement.
>
> **Example:** An author might state that uncertainties in the E_γ values are 0.1 for the strong transitions increasing to 0.5 for the weak ones. After looking at the range of I_γ values listed, and perhaps the spectrum if given, a comment such as "The authors state that the uncertainties in E_γ are 0.1 for strong transitions increasing to 0.5 for weak transitions. The evaluator has assigned 0.1 to transitions with $I\gamma > 10$, 0.3 to transitions with $I\gamma = 5$ to 10, and 0.5 to transitions with $I\gamma < 5$" would be appropriate. Of course an evaluator could choose to make finer divisions if he/she wished.

2. When experimental data are quoted without uncertainties, the evaluator should state explicitly that no uncertainties are given by the authors. In general, the evaluator should not take on the responsibility of adding them. Some exceptions are given in **Notes 1** and **2** below. An attempt should be made to contact the authors to see if uncertainties can be provided. If a paper is relatively recent, authors will usually respond to such requests.

In the case of datasets with γ's, keep in mind that GTOL assigns an uncertainty of 1 keV to E_γ when no uncertainty is given on an E_γ entry, so even for an input dataset with no uncertainties on E_γ, the GTOL output will contain uncertainties on the output E_{level} values. In such cases these uncertainties should be deleted, the energies should be rounded off appropriately, and a footnote added that states something like "No uncertainties are available for the E_γ input. The E_{level} values are from a least-squares fit to the E_γ data with the assumption that the uncertainties are the same for all the E_γ's".

> **Note 1:** The intensity normalization of a decay scheme may sometimes involve a measurement quoted with no uncertainty. As pointed out in **section 1.2.12**, **Normalization**, **Note 1**, **page 12**, the normalization factor should be given as approximate; however, if that measurement is a β or ε feeding, it might be possible to invoke systematics to estimate a reasonable value for the quantity, as discussed above in **section 1.3**, **Systematics, Comment 4, page 17**.
>
> **Note 2:** If there is only one paper in a dataset and no uncertainties are given, and the author cannot be contacted or does not respond to a request, it may be possible to estimate reasonable uncertainties by looking at earlier publications by the same group from the same laboratory using the same general experimental setup where uncertainties have been quoted. The decision as to whether or not this should or can be done in an individual case is at the evaluators' discretion. If done, the procedure followed by the evaluator should be explained.

3. The weighted average program, and all other analysis programs that calculate uncertainties when individual values with uncertainties are combined, such as GTOL, treat the individual uncertainties as statistical in nature. When the uncertainties are known to have a significant systematic component, say due to a calibration uncertainty, the output from the above programs should be modified as necessary. In such cases the adopted uncertainty should be no smaller than the smallest of the input uncertainties.

Note 1: In cases where each of the sources from which values are being taken for a weighted average give separately the statistical and systematic components of the uncertainties, one can take the average of the various sets with just the statistical part included. The smallest of the quoted systematic uncertainties can then be added in quadrature to get the total uncertainty.

Note 2: No result obtained from a weighted or unweighted average program, or by any other method, should be quoted with an uncertainty smaller than the uncertainty(ies) in the calibration standard(s) used to determine the input values.

4. All uncertainties in extracted data should be accounted for, either explicitly or in comments. In addition to the uncertainties quoted in tables, authors occasionally state in table footnotes or in the text that additional uncertainties should be added to get absolute values. These should be taken into consideration.

Note 1: In the case of energies, authors sometimes quote their values relative to some standard, or a set of standards. In such cases the evaluator should check to see if those standards have changed, and if so, and if possible, revise the authors' values to reflect the change in the standards.

Example: In ^{152}Eu decay, an author measured a precise Eγ(411γ) relative to a 1986 value for a ^{192}Ir standard. The result was Eγ=411.126 *3* from their measurement of ΔEγ=56.946 *2* relative to Eγ=468.0715 *12* for the Ir standard. The value for the Ir standard as given in 2000He14 is 468.06885 *26* which gives a revised value of Eγ(411γ)=411.123 *3*.

In cases where the authors state that an addition x keV should be added in quadrature to the uncertainties quoted in their table, it is recommended that this fact be included just as a comment in the source dataset and not added explicitly to the values in the table; however, if a value from this dataset is to be used in Adopted Levels, either by itself or as one value in a weighted average, then the additional uncertainty should be included. This policy preserves the more accurate level spacings in the source dataset, and gives the proper absolute energies in Adopted Levels. In this connection see **Comment 3**, **Note 1** above.

Note 2: In the case of I_γ values, authors sometimes state that in addition to the uncertainties quoted in their table, an additional x% should be added in quadrature to account for the uncertainty in the efficiency calibration. This type of correction, which is independent of E_γ or I_γ, can best be incorporated in NR for decay datasets or in a footnote for reaction datasets. Since transitions close in energy might be expected to have a common correction factor, intensity ratios can sometimes be deduced that are independent of x. The alternative approach to including x, namely including the x% explicitly in each I_γ not only entails more work at the input stage, but rules out the possibility of obtaining the more precise ratios, or at least requires "un-correcting" the relevant I_γ values to do so. As for the E_γ case discussed in **Note 1**, if a value from this dataset is used in Adopted Gammas, the additional uncertainty should be included. In this connection see **Comment 3**, **Note 1** above**.**

5. **Uncertainty in the internal conversion coefficients:** For pure multipolarities, the output from Bricc includes an uncertainty that combines the certainty assumed for the theory and that due to the uncertainty in E_γ. For mixed multipolarities, Bricc will also include the uncertainty in MR. The analysis programs do not add any additional uncertainty.

   Note 1: When a multipolarity is determined by an experimental α value, that value with its uncertainty is what should appear in the CC and DCC fields. Once the multipolarity and MR are determined and Bricc is rerun, the output α value might be slightly different due to roundoff. In such cases be sure to correct back to the input value.

   Note 2: When doing a calculation "by hand" that involves the internal conversion coefficient, the uncertainty in α should be included.

6. When normalizing a decay scheme in which a single γ transition feeds the ground state, so that $I_\gamma(1+\alpha)=100$, the only uncertainty in the absolute intensity of I_γ will be from the uncertainty in α.

7. In the relation between T½ and BE2, for the case of low transition energies with large conversion coefficients, the terms in the factor $[E\gamma^5 \text{ x } (1+\alpha)]^{-1}$ should be treated as a combined term since it may have a smaller uncertainty than the Eγ and α terms treated separately.

8. Uncertainties larger than 25 should, in general, be rounded off, and where feasible, data should be quoted in units such that this convention can be applied. For example, a measured value of T½=250 ps 50 could be given as 0.25 ns 5.

   Note 1: Quantities for which standard units are used in ENSDF should always be quoted in those units. For example, Eγ, Eα, and E_{level} are always quoted in keV, so E_{level}=2560 *250* should not be expressed as 2.56 MeV *25*, but rather as 2.56E+3 *25*. Note that ENSDF does not have standard units for level widths, even though these have units of energy. A convenient unit for giant resonance widths is often MeV, and for level widths from (γ,γ’) is often meV. In these cases the units must of course be specified.

Note 2: In general, the "25" roundoff recommendation applies to final values. Larger uncertainties should be kept for quantities appearing in intermediate steps in a calculation to avoid possible roundoff errors.

Note 3: In a weighted average it is advisable to keep larger uncertainties for the individual input values and just round off the average value itself.

1.5. Resonances

The following is the revised policy statement for inclusion of resonance data adopted by the USNDP at the US-DDP meeting in November, 2010.

1. Charged-particle resonances.

In the source dataset the following quantities should be given as determined in that dataset:

a) Excitation energies in absolute values and not, for example, as S(p)+E(p)
b) Measured resonance energies in a comment record or in a re-labeled field. The coordinate system, lab or center of mass, should be stated.
c) J, π, L
d) Total widths or T½
e) Partial widths in comment records or in re-labeled fields
f) Resonance strength in a comment record or in a re-labeled field
g) Cross sections in comment records
h) Reaction Q value in a comment record
i) Eγ
j) Iγ, relative intensities or branching ratios
k) Gamma-ray multipolarities, mixing ratios, coefficients for angular distribution, angular correlation, polarization, etc.

2. Neutron resonances:

Average resonance neutron capture data should be given. Inclusion of other resonance data is optional, but should be given if information on the bound levels can be extracted.

1.6. L Transfers

1. A brief comment on the method used for obtaining the L values should be given. It is important to distinguish, for example, between L values deduced from a DWBA analysis, and L values based on a comparison of $\sigma(\theta)$ with shapes for levels with known Jπ.

2. Parentheses should be used to denote questionable or uncertain values. Square brackets can be used to indicate an assumed value, such as a value adopted by an experimenter (or by an evaluator) on the basis of known ΔJπ, or a value taken from other work. L values can also be put in square brackets to denote assignments assumed for the purpose of extracting S, or to denote values assumed in order to determine empirical angular distribution shapes for known levels so that L values for other levels can be determined.

> **Note:** The L values should always be quoted as given by the author. The evaluator can then use his/her judgement as to their reliability when incorporating them into Jπ assignments. For example, an author's L=2 which in the evaluator's judgement should be L=(2), should appear as L=2 in the source data set, but as L=(2) if used as a Jπ argument. In such a case a comment is needed explaining that the evaluator feels that the L assignment is tentative. A comment of this type is best given as a footnote/comment in the source dataset rather than in Adopted Levels.

1.7. Spectroscopic Factors

1. The exact label for the quantity given should be defined by using the "LABEL=name" format described in the manual; thus, "LABEL=C^2S".

2. It is recommended that an explicit definition of S be given if there is any ambiguity about what is meant; thus "S is defined by $d\sigma/d\Omega$ (exp)=$NSd\sigma/d\Omega$ (DWBA) with N=xxx"

3. The method for obtaining the scale of S should be given. A comment such as "From DWBA", or "Values are normalized to 3.0 for the 1430 level", should be given.

4. The shell-model, or other orbital designation involved in the transfer should be specified if needed for the extraction of S.

> **Note:** This can usually be done with a general comment such as "L= l, 2, and 3, are assumed to be $p_{3/2}$, $d_{5/2}$, and $f_{5/2}$ except where noted otherwise". An alternative method is to fill in the J field for the relevant levels along with a comment on the J heading such as "J : Value assumed by the authors for the extraction of S". The former approach is preferred when practical.

5. In cases where the J given by an author differs from the evaluator's adopted value, the author’s value of S, which may thus be incorrect, should not be entered in the S field but given only in a comment. The reason for recommending that the incorrect value be given at all, is that a knowledgeable reader can perhaps estimate from the value calculated for the incorrect orbital what the value for the correct orbital would be.

1.8. Jπ

1. **For decay data sets and reaction datasets with γ's.**
Jπ values from Adopted Levels should be given. The introductory section to the Nuclear Data Sheets includes this explanation; however, it it is recommended that such a comment still be given explicitly. The introductory material is not given in every issue of the *Nuclear Data Sheets*, and on-line users may not notice this section. Exceptions to this procedure should be noted. J and/or π values determined in such experiments should be given in comments along with their justification. This is especially important if such values are used as part of the Jπ argument in Adopted Levels

> **Example 1:** Comments such as "γγ(θ) consistent with J=7/2, not consistent with J=5/2, or 9/2", "J=1 from γ(θ)", and "Fed by primary γ from the J=1/2+ capturing state" are appropriate. See also **Comment 4.** below
>
> **Example 2:** If quoting Jπ values given by the authors, some justification is needed. For example, "From xxxxx based on γ(θ) and proposed band structure".

2. **For reaction data sets without γ or Jπ information**.
It is recommended that Jπ values, whether from Adopted Levels or from some other dataset, not be given, unless they are important in explaining some other aspect of the experiment. Some examples are given below.

> **Example 1:** In a dataset where J has not been determined but where the J values used to extract C^2S factors are needed, it might be convenient to give them in the J field rather than in a comment. The source of the J values of course needs to be given. In most cases it would be "From Adopted Levels". See **section 1.7. Spectroscopic Factors**, **Comment 4, Note, page 23**.
>
> **Example 2**: In a dataset such as (γ,γ') in cases where J has not been determined but where the J values used to extract widths are needed, it might be convenient to give those J values in the J field rather than in a comment. The source of the J values of course needs to be given. In most cases it would be "From Adopted Levels".

3. **For reaction datasets without γ data but with Jπ information**.
The deduced Jπ values should be given in the J field along with a comment stating how they were determined.

> **Note 1:** Jπ values that come directly from L values, such as J=L± ½ for single-particle transfer on an even-even nucleus, or J=L in (p,t) on an even-even target, are redundant, and should not be given. In such cases the L value is sufficient.

4 Arguments used in the Jπ assignments in Adopted Levels must be documented in the source

data sets. The following are just a few examples.

Jπ	Argument used in Adopted Levels
a) $3/2^-$	L(d,p)=l gives $1/2^-$, $3/2^-$. 392γ to $5/2^-$ is Ml
b) 1^-	Av. Res. (n,γ) gives 0^-, 1^-. γ to 0^+
c) 3^+	El γ to 2^-. γγ(θ) consistent with J=3, not with J=1 or 2.
d) $(5/2)^+$	L=2, C^2S in (d,p)

In a), the (d,p) data set should contain the L value referred to, with any explanation deemed necessary to justify or explain it. The Adopted Gammas data set should contain the justification for the Ml assignment to the 392γ.

In b), the Av. Res. (n,γ) data set should contain the value deduced in that data set, given in a comment.

In c), enough details on the γγ(θ) experiment should be given in the source data set to justify the conclusion. Briefly, this section should mention the assumptions, that is, what J's for other levels and what δ 's for relevant gammas in the cascade were adopted, and should clearly state which values of J are allowed and which are ruled out. In the above example, it is only necessary to state that γγ (θ) is consistent with J=3, and rules out J=l and 2. Unless J=1 and 2 are specifically ruled out, consistency with J=3 by itself adds nothing to the argument and one would be left with the assignment Jπ=1+,2+,3+ based just on the E1 γ.

In d), the (d,p) data set should contain the L and C^2S values for the level in question, along with a comment justifying the basis for the C^2S argument. For example, "$d_{3/2}$ strength exhausted by known $3/2^+$ levels. C^2S for the L=2, E=xxx level suggests $d_{5/2}$ ". Note that if C^2S were sufficiently large, and the possibility of the level being a multiplet can be ruled out, the argument for ruling out J=3/2 might be considered strong, giving Jπ=$5/2^+$.

1.9. I_γ, $I_{\gamma+ce}$

1. Iγ data should be given as relative values rather than as branching ratios whenever possible. If both relative Iγ and independently determined branching ratios are available, both should be given. The branching ratios can be given in a re-labeled TI field.

> **Note:** If the TI field is already being used, say for $I_{\gamma+ce}$ data, then the branching ratios can be put in comments.

2. For reaction γ's, the projectile energy and the angle at which the quoted intensities were measured should be specified in footnotes on the column headings, unless such information is obvious from the keywords given in general comments.

Note: Relative Iγ values measured under different experimental conditions, such as at a different bombarding energy or angle, should not be combined in the RI field except where an I_γ from level "X" can be deduced from its branching relative to other transitions from level "X".

3. Gamma intensities reported as upper limits are important data measurements and should be included. A comment to the effect that the transition was looked for but not seen could be included. An Iγ given by an author as "weak" should be so noted in a comment or flagged footnote. The latter is preferable since a symbol will appear in the Iγ column and will be more likely to be noticed.

Note 1: It is important to distinguish between the cases where I_γ is not given because it is weak and where it is not given because it is obscured by an impurity (and thus could be strong).

Note 2: A value quoted by an author as <A±ΔA should be entered in the RI field as <A+ΔA along with a comment giving the authors' value of <A±ΔA . Thus I_γ<3.2 *5* should be entered as I_γ<3.7.

4. Unless being used as a re-labeled field, the TI field should be used only if $I_{\gamma+ce}$, rather than I_γ, is the quantity measured or deduced. Two common cases where this occurs are where TI is deduced from intensity-balance arguments, or where TI is deduced by summing measured I_{ce}, such as for an E0 transition or for a low-energy highly converted transition.

Note 1: When both I_γ and $I_{\gamma+ce}$ are known, then of course one should calculate α and deduce the multipolarity. When TI is known but no I_γ is available, then if α is known, the corresponding I_γ should be calculated and entered into the RI field. The uncertainty given for this deduced I_γ should include that in both TI and α. A comment should be given stating that the I_γ comes from TI and α.

Note 2: An I_γ deduced from TI and α may be given in the RI field even when a direct measurement of I_γ is available if the evaluator concludes that the deduced value is more reliable and/or is of higher precision than the measured value. The measured value, with reference, should be included in a comment.

Note 3: When TI, rather than I_γ, is the basic measured or deduced quantity, then the K/T etc., format on the continuation record should be used. K/T, for example, operates directly on TI to generate I_{cek} (via RadList) and the resulting x ray intensities. Note that Bricc outputs K/T etc., if the TI field has an entry, so this caution applies only if the SG record is being generated "by hand". This format avoids including some uncertainties twice, since I_γ, if calculated from TI and α, will already have an uncertainty combined from these two quantities so that I_{cek}, if calculated from I_γ and α_K would double count a portion of the uncertainty.

5. Do not put TI values in the RI field, even if a comment is included to explain what is being done, and even if all the entries are TI values. It is especially important to avoid mixing RI and TI in the same field.

6. The RI and/or TI field should be left blank for transitions that deexcite an isomeric state in the daughter nucleus if the isomeric T½ value is such that the intensities are time-dependent. A comment should be included giving the % feeding of the isomer, and a comment is also needed to explain why the intensities are missing. The intensities can be given in a separate IT decay dataset.

7. $I_{x\text{-ray}}$ and $I_{\gamma\pm}$ data should be given as comments. It is recommended that they be given in the form $I_{x\text{-ray}}/I_{\gamma i}$, where i is the transition to which the Iγ's are normalized. This procedure avoids the necessity of changing the comments if the Iγ's are later re-normalized. It is recommended that the program RadList be run to compare the measured x-ray and I_γ intensities with those calculated on the basis of the adopted decay scheme. If the $I_{x\text{-ray}}/I_\gamma$ or $I_{\gamma\pm}/I_\gamma$ measurements are needed to get the decay scheme normalization, note that RadList can be used in an iterative fashion to deduce NR.

8. Internal conversion intensities used to determine multipolarities should be given. Other situations where they are needed are:

a) I_{ce} ratios measured with a precision of better than about 3%, as well as values for transitions within about 2 keV of the binding energy, can be very useful in checking the validity of the Bricc code.

b) Where no I_γ is given, or where the I_{ce} are more precise, I_{ce} values should be quoted.

c) I_{ce} are needed for E0 and anomalously converted transitions.

9. For transitions whose intensity is given as an upper limit, the intensity, I<A should be converted to I=½A ± ½A for the purpose of calculating quantities that require the intensity of this transition, such as normalization factors, β^- and $\varepsilon+\beta^+$ feedings, or branchings.

> **Note 1:** In a situation where $I_{\beta-}$(gs) is determined to be <6%, the intensity should be expressed as 3% *3* for the purpose of obtaining the gamma intensity normalization. That is, one should set sum TI(gs)=97 *3* and explain what is being done. This procedure is preferable to any of the alternatives, namely setting TI(gs)=l00, or TI(gs)>97. There is no justification for adopting the first alternative, and adopting the second alternative leads to lower limits being given for all the intensities. See the following note.
>
> **Note 2:** The usefulness of the procedure described in **Note 1** depends of course on the value of the limit itself. If $I_{\beta-}$ is known only to be <70%, then perhaps it is not

worthwhile normalizing the decay scheme. If no normalization is adopted in this case, a comment could be given stating what the normalization factor would be for the extreme cases, namely for $I_{\beta-}$=0, and $I_{\beta-}$=70. The intensity of the gs β^- group should still be given as a limit in the β^- listing.

Note 3: I_γ values given as limits should be converted to $\frac{1}{2}I_\gamma \pm \frac{1}{2}I_\gamma$ for the purpose of obtaining β^- and/or ε feedings from intensity imbalances. GTOL treats limits in this fashion. This procedure may lead to some feedings with large uncertainties, but that correctly reflects the state of knowledge of the decay scheme.

10. For transitions whose placements are uncertain, that is, transitions with a "?" in column 80, I_γ should be handled like transitions given as limits discussed above. That is, I_γ=A ± ΔA should be taken as I_γ = ΔIγ = ½(A+ΔA). GTOL treats limits in this fashion.

1.10. Multipolarities, mixing ratios, conversion coefficients

1. For decay datasets, the multipolarity, δ, and α entries should be adopted values. The inclusion of such data is mandatory, an exception being negligibly small values of α. See also **comment 9.** below.

Note: The value of α should be for the adopted Eγ and not for the Eγ given in the dataset of interest, in cases where these are different.

2. In reaction data sets, multipolarity, δ and α should be included only if needed or if measured in that dataset.

3. In any data set in which multipolarity, δ, or α are determined, the bases for such determinations should be stated. The sources of data used by the evaluator to determine multipolarities, such as γ(θ), α_k(exp), should be specified. In the case of α_k(exp), the normalization used to put relative I_γ and I_{cek} values on the same scale should be given. Normalizations used by authors may need to be revised if newer values for the standards they used become available. Multipolarity assignments from ce data should be those of the evaluator based on the output from Bricc. Multipolarities deduced by the authors (or by the evaluator) on the basis of γ(θ) to be "stretched" should be so noted. Comments such as "ΔJ=1 from γ(θ)", or "Stretched Q from γ(θ)" etc., are recommended. It is especially important that such conclusions be mentioned if they are used in Adopted Levels, Gammas.

4. Angular correlation or angular distribution data determine only the L component of the gamma character, thus mult=D, D+Q, etc. Further assumptions are needed to establish Δπ. These assumptions should be stated when D is converted to M1, or D+Q to M1+E2, etc., and it is recommended that this step be taken only in Adopted Gammas and only if needed, say for a Jπ assignment or for the calculation of BE2W, BM1W, etc. See **Adopted Gammas**

Datasets for further discussion. If the polarization of a transition has been measured, then of course Δπ can be determined and mult= M1, E1, etc., as determined by an author should be given.

> **Note:** If T½ is known, RUL can sometimes be invoked to eliminate one of the multipolarity choices, for example Q=M2, or D+Q=E1+M2 when δ is known. This should be done in the source datasets using adopted T½ and δ data.

5. The entries in the multipolarity (M), mixing ratio (MR), and conversion coefficient (CC) fields should be mutually consistent. In particular, the following guidelines should be followed.

a) If a single multipolarity is adopted, the MR field should be blank. If only a limit on δ is available, there are two options.

> **i)** Give the dominant component in the M field, with the corresponding CC, and give the MR limit in a comment.
>
> **ii)** Give both components in the M field and give the δ limit in the MR field. In this case, CC should be the value corresponding to ½ δ(max) with an uncertainty chosen to overlap the δ=0 to δ=δ(max) range.
>
> **Note:** Option i) is recommended when, in the evaluator's judgement, the admixed component is likely to be smaller than the experimental limit; thus, for E2+M3 with an experimental limit of δ<0.5, since it is unlikely that an M3 component would compete so strongly with an E2, the value of δ is likely to be much smaller. In this case the multipolarity should probably be entered as E2, with a comment stating "δ(M3/E2) < 0.5 from xxxx". Since E2 and M1 can compete strongly, MI+E2 with δ <0.5 should probably be retained as a mixed multipolarity entry.

b) Same as a) but with a lower limit on δ. In this case consistency with known Jπ values in the level scheme needs to be considered. Thus if mult=M1+E2 with δ>1.5 is deduced, but placement in the level scheme requires ΔJ=2, option **i)** should be chosen.

c) If two multipolarities are given but no δ is known, the corresponding α value should be the value calculated as in **9. a)** below.

d) If the multipolarity field contains more than two multipolarities, for example, E1+M2+E3 or E0+MI+E2, the relevant mixing ratios should be given on continuation records. See the following section on E0 transitions for the format for the second case. For the first case, the format is MR(M2/E1)=... and MR(E3/M2)=.... Bricc will calculate α and enter it into the CC field and if T½ is known, RULER will calculate BE1W, BM2W, and BE3W and enter them on a continuation record.

e) If δ overlaps zero or infinity, the corresponding small multipolarity component should be in parens.

> **Example:** For mult =M1+E2 with δ=0.3 *4*, the multipolarity should be entered as M1(+E2). For δ>10, the multipolarity should be entered as E2(+M1), except as noted in **5. b)** above.

6. The mixing ratio notation, M1+x%E2 occasionally used by authors should be converted to δ. In doing so be sure to use the constraint that the percentage of the two values must equal 100. Thus, M1 + 10±8% E2 gives δ^2=10/90 with a maximum of 18/82 and a minimum of 2/98, or δ^2=0.11 *+11-9* and thus δ = 0.33 *+14-19*. See **section 3.3. Iγ**, **Comment 3, Example**, **page 43** for another case.

7. The notation mult=M1,E2 is not the same as mult=M1+E2. The notation M1,E2 describes the case where the data are consistent with pure M1, pure E2, or a mixture. The notation M1+E2 describes the case where the data rule out pure M1 and pure E2. The designation M1(+E2) is an intermediate case where the experimental data overlap the M1 theory value but not the E2 theory value.

8. If αk, etc., data or conclusions from such data are quoted, the bases for the values used should be given. If from relative I(ce) and Iγ, the basis for the normalization of the relative scales should be stated. Be sure that the multipolarity for any transition used in the calibration is independently established.

9. In cases where Iγ is known, and internal conversion is significant, but the multipolarity is not known (apart from level scheme considerations), and TI is otherwise unobtainable and needed, the following procedures can be invoked.

a) If ΔJ and Δπ are known, one can enter mult=[E1], [M1,E2], etc., in the multipolarity field and choose α accordingly. For mult=[MI,E2], one should enter α=½[α(M1)+α(E2)] and Δα=|α - (M1)| = |α - (E2)|.

b) If ΔJ and/or Δπ are not known, one can still follow the procedure described in **a)** and set, for example, mult=[D,E2] (or mult=[E1,M1,E2]). Mult=M2 or higher are assumed here to be less probable, but of course could be included.

The usefulness of either **a)** or **b)** depends of course on the range of values for the quantity 1+α for the assumed multipolarities.

> **Note 1:** If ΔJ=0 or l, Δπ =no, excluding the transitions J=0 to J=1 or 1/2 to 1/2, then mult=[MI,E2] rather than mult=[M1] or mult=[E2] should be adopted, unless there are good arguments for believing that one of the two possible multipole components dominates. α for mult= M1,E2 is always "correct", although it may have a large

uncertainty, whereas [M1], for example, may lead to misleading conclusions. The possible large uncertainty in α when δ is not known correctly reflects the state of knowledge concerning the total intensities.

Note 2: The use of the mult=[ ] convention in source datasets should be restricted to cases where the internal conversion is significant for the purpose at hand, such as determining intensity balances for β^- or $\varepsilon+\beta^+$ feedings or for estimating total transition intensities. Avoid filling up the multipolarity fields with square bracket multipolarities that are not needed.

10. The experimental αk, etc., as well as ce ratios that are used to determine multipolarities should be given. As mentioned above, values measured with a precision of better than about 3%, as well as values for transitions within about 2 keV of the binding energy, can be very useful in checking the validity of the Bricc code.

11. Since the CC field cannot accommodate asymmetric α values, such values must be symmetrized. In such cases the experimental asymmetric value should be given in a comment.

12. As discussed in **section 1.4, Uncertainties, Comment 5, Note 1, page 19**, when a multipolarity is determined from a measured α, that value of α is what should appear in the CC field.

13. Note the distinction between ( ) and [ ] for multipolarities. These are discussed in the introductory material to the *Nuclear Data Sheets*. Parentheses are used when there are some experimental data, but the data are not conclusive. The square brackets are used to denote a value deduced solely from level scheme considerations.

Note: For the case where mult=D+Q is determined from angular correlation or angular distribution data, and the level scheme is used to assign M1+E2 rather than E1+M2, then the multipolarity should be in parentheses, that is, mult=(M1+E2), with a comment stating something like "Mult: D+Q from γ(θ) in xxx, Δπ =no from the level scheme". Square brackets are not appropriate for this case, since the level scheme argument forms only part of the assignment. Also, the fact that mult=D+Q can be used as a strong argument for ΔJ<2, whereas mult=(M1+E2) is a weak argument for such a conclusion.

14. Do not show α as a lower limit. This would result in TI=Iγ(l+α) appearing incorrectly as an indefinite quantity. This situation can arise for transitions that have an E0 component. The basic data are usually a measured I_{cek} and an upper limit on Iγ, say <X, which lead to $\alpha_k >$ I_{cek}/X and α >Y. TI=Iγ(1+α) then becomes (<X)(1+ >Y). The recommended procedure for obtaining TI will depend on the relative magnitude of I(ce) and the Iγ limit. For I(ce)>>X, one should give TI=I(ce) + ½X with an uncertainty in X chosen as ½X. For X>>I(ce), TI<[X+I(ce)] is an appropriate choice. For the intermediate case, the first alternative is recommended

> **Note:** For a transition adopted as pure E0, then of course TI=I(ce) and there will be no entry in the RI field.

15. Angular correlation and angular distribution data usually give two solutions for δ. Unless one of these solutions can be ruled out, both should be given in a comment and no entry should be given in the MR field.

> **Note:** If neither solution overlaps zero or infinity, then the multipolarity can be assigned as, say, D+Q; otherwise the multipolarity should be given as D,Q.

1.11. E0 transitions

RULER should be run to calculate the absolute monople strength, ρ^2(E0), for E0 transitions. In order to do this, the branching of the E0 component from a given level, along with the level half-life, must be known. See **Appendix D** for relevant formulas.

In the general case of an E0+M1+E2 transition one needs the mixing ratio MR(E2/M1), given by δ, and the ratio I(E0)/I(E2), given by the square of the mixing ratio denoted by MR2(E0/E2). This latter ratio is usually denoted in the literature as q^2(E0/E2) and it includes internal conversion electron and electron-positron pair emission. Since the most common measurement of the relative E0 and E2 intensities is through K-shell internal conversion, and in order to keep the number of continuation record quantities to a minimum, all the I(E0)/I(E2) ratios should be given in terms of K-shell internal conversion, MR2K(E0/E2)=q^2_k(E0/E2)=I_K(E0)/I_K(E2). RULER will calculate the full intensity ratio from this quantity. These quantities should be given on a continuation record using the format "MR(E2/M1)=" and "MR2K(E0/E2)=". From these two quantities, RULER will calculate ρ^2(E0), BE2W, and BM1W and put them on continuation records, and Bricc will calculate the total conversion coefficient, α, and enter it in the CC field on the gamma record.

The following material on Jπ=0+ to Jπ=0+ and $J_i = J_f$, $J \neq 0$ transitions is taken from Ref [1] in **Appendix D** where the situations likely to be encountered are discussed. In addition to the following examples involving intensity ratios, as pointed out in the above reference, ρ^2(E0) can also be determined from the form factor in inelastic electron scattering. This method has been used for E0 transitions in some light nuclides.

1. Jπ=0+ to Jπ=0+ with no intermediate level:

Nothing needs to done. The transition has a branching of 100%, so if T½ is known, RULER will calculate ρ^2(E0) from the expression

$$\rho^2(E0) = \frac{\ln 2}{\Omega(\mathrm{E0}) \text{ x } T_{1/2}}$$

and enter it on a continuation record. Ω(E0) is an electronic factor.

2. **Jπ=0+ to Jπ=0+ with an intermediate 2+ level:**

The ratio of the E0 K-shell conversion intensity to that of the competing Jπ=0+ to Jπ=2+ transition, MR2K(E0/E2), must be known. From this ratio RULER will calculate the total E0/E2 intensity ratio, including internal conversion electron and electron-positron pair emission, and then calculate ρ^2(E0) and BE2W.

Example 1:
I_k(E0) and I_k(E2), or their ratio, are known. This provides a direct determination of MR2K(E0/E2). This value should be entered on a continuation record. RULER will calculate ρ^2(E0) and BE2W, and Bricc will calculate α.

Example 2:
I_k(E0) and Iγ(E2), or their ratio, are known, and MR2K(E0/E2) has been deduced using the internal conversion coefficient α_k(E2). As in **Example 1**, MR2K(E0/E2) should be entered on a continuation record.

Example 3:
I_π(E0) and I_π(E2), or their ratio are known, where π refers to electron-positron pair emission. In this case, one has

$$I_K(E0) = \mathrm{I}_\pi(E0) \text{ x } \frac{\Omega_K(E0)}{\Omega_\pi(E0)} \text{ for E0, and}$$

$$\mathrm{I}_K = I_\pi \text{ x } \frac{\alpha_K}{\alpha_\pi} \text{ for all other cases. Thus we have}$$

$$MR2K(E0/E2) = \frac{I_\pi(E0)}{I_\pi(E2)} \text{ x } \frac{\Omega_K(E0)}{\Omega_\pi(E0)} \text{ x } \frac{\alpha_\pi(E2)}{\alpha_K(E2)}$$

The electronic factors can be obtained from Bricc to deduce MR2K(E0/E2). A simple modification of this expression can be used if one has measurements of I_π(E0) and I_k(E2), or I_k(E0) and I_π(E2).

Example 4: I_L(E0) and I_L(E2), or their ratio are known. In this case one has

$$MR2K(E0/E2) = MR2L(E0/E2) \text{ x } \frac{\alpha_L(E2)}{\alpha_K(E2)} \text{ x } \frac{\Omega_K(E0)}{\Omega_L(E0)}$$

As in example 3, Bricc contains the Ω factors, and simple modifications of this expression can be made for any combination of shells for the E0 and/or the E2 components.

3. Jπ=0+ to Jπ=0+ with two intermediate 2+ levels:

Example:

I_k(E0) and I_k(E2'), or their ratio, are known, where E2' denotes a transition other than 0^+_1 to 2^+_1, and the relative intensities of the two E2 transitions are known.

$$I_k(E2) = \frac{I_\gamma(E2)}{I_\gamma(E2')} \text{ x } \frac{\alpha_k(E2)}{\alpha_k(E2')} \text{ x } I_k(E2')$$

The deduced MR2K(E0/E2) can then be given. Note that this approach can be extended to cases where there are more than two intermediate 2+ levels, with the transition to the first 2+ level always being the reference.

4. Jπ to Jπ, J≠0:

Example:

Z=64, Eγ=586.27 *7*, Iγ=14.5 *3*, α_k(exp)=0.0202 *21*, δ(E2/M1)=-3.05 *14*, α_k(theory)=0.01439 *21* (M1), 0.00764 *11* (E2), and 0.00829 *13* (for δ=3.05 *14*).

These data give I_{cek}(exp)=0.293 *31* compared with I_{cek} (from γ's)=0.120 *3* and thus an E0 component of I_k(E0)=0.173 *31*. The E2 contribution is Iγ x α_k(E2) x $\delta^2/(1+\delta^2)$=(14.5 *3*) x (0.00764 *11*) x (0.903 *8*)=0.100 *3*. One then gets MR2K(E0/E2)=1.73 *31*. The evaluator would enter "MR(E2/M1)=3.05 *14*$ MR2K(E0/E2)=1.73 *31* " on a continuation record.

> **Note:** For the case where I_K(exp) > I_K(M1 or E2) but the lower uncertainty overlaps, if the data are accurate and reliable one can give an upper limit on the E0 component, and if MR(E2/M1) is known, then an upper limit on ρ^2(E0) can be given.

1.12. g-factors, electric and magnetic moments

1. Values of the magnetic dipole moment, μ, and electric quadrupole moment, Q, should be taken from the evaluation 2011StZZ (or updates when available) and entered directly into Adopted Levels. There is no need to repeat these values in source datasets. Values of μ or Q that appear in the literature after the cutoff for values in 2011StZZ should be added.

> **Note 1:** If the method of determining μ depends on T½, and if the value adopted by the evaluator differs from that used in 2011StZZ, then a correction should be made if possible. If the value cannot be readily corrected, then a comment should be included giving the T½ used in the 2011StZZ evaluation.
>
> **Note 2:** For new data, if the values are of comparable precision to those listed in 2011StZZ, it is recommended that the evaluator contact the author of 2011StZZ to

see if intermediate recommendations are available.

2. g-factor data should be given in the appropriate source data sets. These values should be corrected, where necessary, for the adopted T½. When corrected, a comment such as "g: For T½ =xxx The authors report g =xxx for T½ =xxx". A comment is also needed stating whether or not the diamagnetic correction has been applied (if the data are accurate enough to be affected by this correction).

2. Adopted Levels Datasets

2.1. General

1. All distinct levels that are observed in any of the individual data sets, and that the evaluator feels are firmly established, should be included in Adopted Levels. Uncertain levels, that is, levels shown with a "?" in one or more of the individual data sets, can be included or not included at the evaluators discretion. Neutron and proton separation energies should not be included as levels. Isobaric analog states (resonances) should be included as should proton and neutron resonances as discussed in **section 1.5. Resonances,** p. 22.

> **Note 1:** To avoid the introduction of "extraneous" levels, the calibration and general trend of energies compared with adopted values should be checked for each data set. Systematic shifts of energies in one or more data sets should be corrected for when the energies from such data sets are used in obtaining the adopted value. This procedure will help avoid the incorrect association of a level in one reaction with a level in another reaction based only on the energy difference and will help to ensure that the energy adopted for a level seen in only one reaction requiring an energy shift is as consistent as possible with other levels.

> **Note 2:** When levels from two (or more) reactions lie close in energy (that is, the values agree within the uncertainties) and the evaluator chooses to adopt both (or all) levels, the justification for assuming that the levels are distinct should be given, unless obvious from XREF or from other adopted level properties. Consider the following cases.

a) E=5000 *10*, J =3/2+ and E=5010 *10*, J =5/2+ are known from particle reactions, and E=5005.3 *2* is known from a gamma experiment; however, it is not known to which of the two reaction levels this level corresponds and there is no evidence to suggest that it is a separate distinct level. The reaction levels should be adopted, with a comment on each stating that the more accurate value of 5005.3 *2* probably corresponds to one of the two Adopted Levels. Note that there is no unambiguous way to include the accurate energy as an adopted energy. The evaluator should not adopt three levels, unless there is definite evidence that the gamma-deduced level is distinct from the others.

b) E=596.7 *5* with J =0+,1,2 and E=597.1 *3* with J =l+,2,3 are known to be different levels, and l(p,d)=2, leading to J =l-,2-,3- with E=598 *2* is also known. Unless there is evidence to suggest that the (p,d) level is distinct, only two levels should be adopted, with a comment on each stating that J =l-,2-,3- from l(p,d)=2, for one (or possibly both) of the levels.

2. Make use of the XREF entries to avoid unnecessary comments

> **Example 1:** A comment such as "seen only in (d,p)" is not needed since XREF should already convey that information. An exception could arise, however, if the evaluator wishes to emphasize some doubt about the level.
>
> **Example 2:** XREF can convey the "one level corresponds to many levels" situation so that comments that repeat only this information are not needed. Note, however, that comments such as "L(d,p)=l for E=3450", given for two or more Adopted Levels to which the (d,p) level could correspond, are still needed.

3. Important comments on level properties which appear in source data sets should be repeated in the Adopted Levels data sets. Comments such as "doublet", "possible contaminant", "not resolved from X", if important in a source data set, are usually just as important in Adopted Levels.

4. If the evaluator adopts a Q value, say Q^-, that is different from the value given in the most recent mass adjustment, the mass adjustment value should be given in a comment for comparison. If the mass links are not too complicated, it may be possible to adjust the other entries on the Q record to reflect the change in the Q^- value. If such is the case, and if the change in Q^- is significant, that is, considerably outside the limits given by the mass adjustment, then giving the adjusted S(n), S(p), and Q(α) values would be a valuable contribution. Whether this is done or not, however, is left to the discretion of the evaluator.

> **Note:** In cases where it is not feasible to attempt a readjustment, a comparison between the mass adjustment value and the adopted value at least allows the reader to judge qualitatively what the effect on the other Q values may be.

5. BE2 and T½ should not both be given as adopted properties of a level. These are equivalent pieces of data (if all quantities needed to convert from one to the other are known), and our policy is to give T½ (See **Note 1:** for an exception). The best BE2 value will then, by definition, be that deduced from the adopted T½ value and the adopted γ properties. We do not give this BE2 value explicitly.

> **Note 1:** BE2 should be given if T½ is not known and cannot be deduced from BE2, for example if the γ branching is not known.
>
> **Note 2:** If T½ and BE2 are both determined for a given level but the γ branching is not known independently, then T½ and BE2 can be combined to deduce the branching. T½ would be given and the deduced branchings would appear in Adopted Gammas with a comment such as “Deduced from T½=xxx and BE2=yyy”.

6. When giving the source of an adopted quantity, include the name of the dataset. The data sources are much easier to locate with this information. The method and keynumber are optional except for adopted T½ values where the method should be given.

> **Example:** A T½ value extracted from BE2 should be stated as such, thus “T½: From BE2 in Coul. Ex.”.

2.2. Excitation energies

1. The source for excitation energies should be given. This source is usually best given as a footnote on the E(level) heading. A statement such as "Unless noted otherwise, excitation energies for levels connected by gamma transitions are taken from a least-squares fit to the adopted gamma energies. Other excitation energies are based on best values from all available reactions" is recommended. The introductory section to the Nuclear Data Sheets includes this explanation; however, it is recommended that such a comment still be given explicitly. The introductory material is not given in every issue of the *Nuclear Data Sheets*, and on-line users may not notice this section. Exceptions to this procedure should be noted.

2. Uncertainties should be included where available. See **section 1.4. Uncertainties, page 19.**

3. In cases where the presumed gs and a long-lived isomer are nearly degenerate, the mass adjustment should be consulted to see if it has information regarding the relative order of the two states. Sometimes a plot of the energy separation of these states in adjacent nuclides can suggest which is likely the gs. in the nuclide of interest. If the order cannot be established, then the notation 0+x and 0+y should be used for the two states.

2.3. J,π

1. Assignments should be based on the fewest and best arguments. There are two main advantages to this "fewest and best" approach. First, the Jπ arguments are easier to read and to follow when redundancy is eliminated. Second, alternate unneeded arguments can be used to build up systematics.

> **Example:** Consider the assignment of 1^+ to a level based on the arguments "M1 γ to 0^+. Logft=4.4 from 0^+". Either argument by itself is sufficient. If the multipolarity argument alone is used, the logft value is then "freed up" and can be added to the base of values from which the logft arguments are derived, thus helping to build up confidence in the application of such systematics to cases where other strong arguments are not available.
>
> **Note:** The above refers to strong arguments. For levels where only weak arguments

are available, then the more arguments that can be given, the more confidant one becomes in the assignment; however, no combination of weak arguments constitutes a strong argument.

2. "Direct" measurements of J (atomic beam, etc.) should be referenced directly and the method should be stated, thus "atomic beam", "NMR". In many cases the reference 1976Fu06* is still a useful source. Note that these methods give J only. A separate argument is needed for π .

* Gladys H. Fuller, Journal of Physical and Chemical Reference Data **5**, 835, (1976).

3. Arguments should be detailed enough to convince the ENSDF user that the assignments are reliable and also to allow the reader to judge what the consequences would be if new data were to become available. See also **4.** below.

Example 1: The argument "From (α,xn)" is not of much use. Statements such as "Excit. in (α,xnγ)", and/or "γ(θ) in (α,xnγ)" are needed. See **5.** below.

Example 2: Consider an argument for the assignment Jπ=2^-,3^- expressed as "L(d,p)=l gives 0^- to 3^-. γ to 4^-". If the γ transition were to be subsequently determined as M1, it would follow that Jπ would be 3^-. If the argument had been given only as a general statement such as "From L value in (d,p) and γ feeding", the consequences of the new piece of evidence would not be transparent.

4. Gamma-property arguments should be specific; thus "Ml γ to 2^+", "γ 's to $3/2^+$,$5/2^+$", etc. The gamma energy is optional, thus "326γ to 2^+ is Ml", etc., and is of most use in complex level schemes where the level referred to as being fed may not be obvious. The vague statement "Jπ is based on γ-decay modes" is not of much use to the reader. Note that Jπ values and γ-ray multipolarities referred to in these comments should be adopted values; thus "Ml γ to ($3/2^+$)", "(E2) γ to $(4)^-$" etc.

Note: Include target/parent Jπ when the target is not even-even. For example, "logft=5.4 from $1/2^+$", or "L(p,t)=2 from $9/2^+$".

5. For arguments that are common to several levels, there are two approaches that avoid writing the same full argument for each relevant level.

a) The argument can be written as a flagged footnote with the flag attached to the assigned Jπ of the relevant levels. This approach improves readability. For example, the footnote could state "From (α,xnγ) based on excit. and γ(θ)", or "Member of band X based on energy fit and inertial parameter".

b) The argument can be put as a footnote on the Jπ heading itself. For example, such a footnote could state "Assignments from (α,nγ) are based on excit and γ(θ). Assignments from (d,p) are based on L values and analyzing power". Then for the relevant level one need state only "From (α,nγ)" or "From (d,p)". This option is particularly useful in cases where the argument is long.

6. Jπ arguments for two or more levels can be linked if they are interconnected in such a way that giving separate arguments for each level could be awkward, or could give the appearance of circularity.

> **Example:** Consider the β^- decay of a parent level with known Jπ=7^- to a daughter level A followed by a γ cascade with known multipolarities through levels B and C to a level with known Jπ=2+, thus $7^-(\beta^-)$A(Ml)B(El)C(E2)2+. The argument "Logft=5.1 from 7^- and the Ml-El-E2 γ cascade to 2^+ uniquely establishes Jπ(A)=6^-, Jπ(B)=5^-, and Jπ(C)=4^+". This argument can be given for one of the levels, say C, and then for the others, one can simply say "Jπ: See C level".

7. An L=0 component in a particle transfer reaction in which S=0 can be assumed leads to ΔJ=0, Δπ =no even if other L components are present. The same is true of an E0 component in a gamma transition.

8. Jπ arguments for the ground state of an even-even nucleus are not needed. A few nuclides have been proven to have J=0 from the absence of alternating intensities in the spectrum of homonuclear diatomic molecules. See 1976Fu06 for details (citation given in **Comment 2**. above), but for ENSDF, Jπ=0+ for even-even ground states is accepted without an argument. An argument such as L(α,α')=0 from 0+ merely shifts the burden of proof from the product nuclide to the target nuclide. The argument "No hyperfine structure seen in optical spectroscopy measurements" proves only that J=0 or that μ (or Q) is very small.

9. Maintain consistency between source data and conclusions. For example, L(p,t)=2 (S=0 assumed) from an even-even target gives J =2^+, not $(2)^+$ or $2(^+)$. That is, if the L value is considered to be a strong argument for J, then it is also a strong argument for π . Similarly, if the argument is not considered strong for J, then it should not be considered strong for π; thus, L(p,t)=(2) gives Jπ=(2+).

> **Note:** A reaction such as (α,d), with a measured L value, can of course be used as a strong argument for π, namely, $\pi=(-1)^L$, even though J is determined only as J=L-l, L, or L+l.

10. Expressions such as "preferred", or "consistent with" are not strong arguments. Avoid these expressions since they leave open the question of whether other alternative J values have been ruled out. These expressions are of course valid for weak arguments.

11. Configurations should be given, but note that "Conf=3/2[521]" is not a valid argument for Jπ . All that this argument accomplishes is to shift the burden of proof from establishing Jπ=$3/2^-$ to establishing conf=3/2[521]. The configuration is usually deduced from Jπ, not vice-versa, although of course sometimes the reverse is true, and sometimes the same argument for Jπ can be used to assign the configuration*.

* A measurement of μ can sometimes determine both Jπ and the configuration.

> **Note 1:** The determination of L and analyzing power in a transfer reaction might give Jπ=$1/2^-$, and depending on the mass region it might be reasonable to assign this level as a $p_{1/2}$ orbital, but the Jπ argument should be "From L and analyzing power in (d,p)", not "From conf=$p_{1/2}$". The configuration should normally be treated as a separate data type from Jπ and put on a continuation record. Comments on "Conf" should also normally be given separately from comments on Jπ.

> **Note 2:** In the deformed regions, the cross sections and cross section ratios, for example in (d,p) and (d,t), can often determine directly the combination JπK[xxx], rather than just Jπ, for example, $5/2^-3/2[521]$, rather than just Jπ =$5/2^-$ by itself. In such cases, the configuration should be included in the Jπ argument.

12. Do not use multiply placed transitions in Jπ arguments unless the connection with the level in question is definite.

> **Note:** A multipolarity determined for a multiplet will not necessarily be the correct multipolarity for each member of the multiplet. See **section 1.2.7. Multiplets, Comment 3, page 8**. If a transition that is multiply placed is definitely established as being connected with the level in question, then the Jπ of the daughter level can be used in a Jπ argument in the usual way, that is "γ to $3/2^+$" for example.

13. When the Jπ choices are limited to three or fewer, it is recommended that they be spelled out rather than given as a range; thus Jπ=5/2-,7/2-,9/2- rather than Jπ=5/2- to 7/2-. There is less chance of values being misinterpreted when they are written out completely.

> **Note:** In many cases, the extra space required (which is the only good argument for quoting J values as a range) is not significant.

14. RUL is an argument for multipolarity, not for Jπ.

2.4. T½*

1. All comparably precise T½ values should be summarized here along with the source datasets from which they come. Details of the measurements are not needed since that information should appear in the source datasets.

2. All values should be given at the 1σ level. Authors measuring half-lives sometimes quote uncertainties as 2σ or 3σ. These uncertainties need to be divided by 2 and by 3, respectively, before averaging with the other values.

* A valuable resource on treating half-life data is the IAEA report **INDC(NDS)-0635**, *Proposed procedures/guidelines for ENSDF half-life evaluations (ground states and long-lived isomers)*, co-authored by A.L. Nichols and B. Singh.

2.5. Other Level Properties

1. When branching modes are given, for example "%IT=xxx", the bases for the values can be given here or in the source data sets, with a cross reference to the location of the arguments. There is no need to repeat the arguments, but they must appear in one place or the other. Also, all possible modes of decay should be accounted for, unless the reason for omitting a mode is obvious.

> **Note:** In a case, for example, where one has "%ε+% β^+=99.0 *1*; %IT=1.0 *1*" but β^- is also energetically allowed, there should be a comment explaining why the β^- branch is considered negligible; thus, for example, "%β^- is negligible since the only available decay branch has ΔJ=2, Δπ =yes, for which, from $\log f^{1u}t>8.5$, one derives % $\beta^-<1\times10^{-4}$". An experimentally determined limit of this magnitude should, of course, be included explicitly in the branching statement. For more obviously negligible branches such as for a case where the only available branch has ΔJ=4, one can state simply "ΔJ=4 for possible β^- branch so % β^- is negligible".

2. BEλ values should be included in Adopted Levels in cases where the T½ is not independently known and cannot be calculated from the BEλ due to missing information, such as a ground-state branching ratio.

3, Sources of data should be stated unless obvious, that is, unless there is only one keynumber associated with the dataset. General comments are usually sufficient; thus, "From X unless noted otherwise" or "Weighted average of values from A, B, and C".

3. Adopted Gammas Datasets

3.1. General

1. For gamma records, all available first-card data should be included.

2. Discrepant data should not be adopted. The data chosen for Adopted Levels should be self consistent.

> **Example:** If a gamma multipolarity disagrees with the adopted ΔJπ, and the Jπ are considered well established, the discrepant multipolarity should not be adopted, and the discrepancy should be pointed out in a comment. It is recommended that a flagged comment be used so that a footnote symbol will appear in the multipolarity field.

3.2. Eγ

Sources for all data should be stated. This can usually be done with a general footnote on the column heading.

> **Example 1:** "Except where noted otherwise, the Eγ quoted to tenths of keV are from β^- decay. The Eγ quoted to the nearest keV are from reaction data".
>
> **Example 2:** "Except where noted otherwise, Eγ from levels below 6570 are weighted averages of data from (α,xnγ) and (HI,xnγ). Eγ for higher levels are from (^{48}Ca,xnγ)".

3.3. Iγ

1. Sources for all data should be stated. This can usually be done with a general footnote on the column heading. See examples under **Eγ** above.

2. As stated in the introductory section to the Nuclear Data Sheets, the Iγ should be given as relative photon branching from each level. Any deviation from this policy, such as quoting branching ratios in %, should be stated. The strongest photon branch should be set to 100 except in the following cases.
   - **a)** The strongest transition is an unresolved multiplet.
   - **b)** The strongest transition is given as an upper limit.

Note 1: An uncertainty should be included in the normalization value of 100 if there is an uncertainty given for the original intensity, unless there is only one transition deexciting the level, in which case the uncertainty has no meaning and should not be given.

Note 2: Iγ for a multiply-placed transition where the intensity has not been divided should be given as a limit in the RI field, thus Iγ, given by the authors as A±ΔA, should be entered as Iγ<A+ΔA) for each placement, and the value Iγ=A±ΔA should be given in a comment.

Note 3: For a level with a single deexciting transition, Iγ=100 should be entered in the RI field. This has no meaning as a relative number, but some non-network programs require a value even in this case.

3. When calculating averages to obtain the relative branchings, take note of cases where the source intensities are given as % branchings, since the uncertainties in these data are correlated.

Example: Consider the case of two transitions, γ_1 and γ_2 and one wants to get a weighted average of the intensity ratio. Some sources may provide relative intensities, and others may quote % branchings, for example, $I_{\gamma1}$=42 *3* and $I_{\gamma2}$=58 *3*. Here the uncertainties are correlated by the constraint that the sum must be 100, thus the ratio is 42/58 with a maximum of 45/55 and a minimum of 39/61 giving $I_{\gamma1}/I_{\gamma2}$ =0.72 *+10-8* rather than 0.72 *7* if the uncertainties were uncorrelated. The correlated uncertainties must be used when this result is averaged with other data. See **section 1.10. Multipolarities, etc., Comment 6, page 28** for another example.

Note: For the case of two transitions, the most straightforward way to get the branching is to average the ratio. Suppose one gets $I_{\gamma1}/I_{\gamma2}$=0.265 *10* from several sets of values. One then has the choice of setting $I_{\gamma1}$=26.5 10 and $I_{\gamma2}$=100, or assigning an uncertainty to the stronger transition and subtracting that fractional uncertainty from the other branch, thus, for example, $I_{\gamma1}$=26.5 *6* and $I_{\gamma2}$=100 *3*. The second approach could be used to reflect the accuracy with which the best relative value for the stronger transition is known, if that is the evaluator's preference.

4 For a transition that has no measured Iγ, or for which just a limit is known, TI should be given, if available. The most common case would be for an E0 transition or for a low-energy transition for which I(ce) but no Iγ or α is available. TI should be given in the same scale as the Iγ values for other transitions in that dataset.

3.4. Multipolarities, mixing ratios, conversion coefficients

1. Sources for all data should be stated and all assignments justified.

> **Example:** When a multipolarity is based on measurements that yield only D, Q, D+Q etc., such as γ(θ) or γγ(θ), and M1+E2 rather than El+M2 is adopted, the basis for this choice must be stated.

2. See **section 1.10. Multipolarities, etc., Comment 5, page 28** for requirements on consistency among the multipolarity, δ, and α entries. α is not needed for transitions with mixed multipolarity and unknown δ, even though such values may have been used in a source data set.

3. The relation between BE2 and T½ allows δ (and/or α) to be deduced in cases where BE2 and T½ are independently known, and the ground-state branching is known. Conversely, the ground-state branching can be deduced if all the other quantities are known.

4. γ(θ) and γγ(θ) lead, in general, to two solutions for δ. Both should be accounted for. In particular, if it is not known which is correct, then both should be put in a comment. Do not put one value in the MR field and the alternate value in a comment.

5. In addition to the use of square brackets as discussed in **section 1.10. Multipolarities etc., Comment 9, page 28**, this convention is also useful in cases where the multipolarity has not been measured, but ΔJπ is known and you wish to show the multipolarity because you are giving, for example, a reduced transition probability.

> **Note 1:** Do not assign mult=[xxx] just because the multipolarity can be deduced from the level scheme or simply because this convention may have been used in a source dataset.
>
> **Note 2:** For the case ΔJ=2, assigning mult=[E2] is preferable to stating "BE2W=xxx if mult=E2". since an entry in the multipolarity field will catch a reader's attention.
>
> **Note 3:** For the case ΔJ=0 or 1, assigning mult=[M1,E2] or [E1,M2], if allowed, is probably not worth doing since only upper limits can be deduced for the BEλW or BMλW.

6. When making the assumption that, for example, mult=D is probably E1, the E1 should be put in parens.

> **Note:** This step should not be taken unless necessary. An assignment of mult=D is

strong, whereas that of mult=(E1) is weak, and for a Jπ assignment the fact that a transition is known to be dipole may be of more use than assuming it might be E1. One case where assigning D as (E1) might be necessary would be where $I_{\gamma+ce}$ is needed and α is significant.

3.5. Reduced transition probabilities

1. Reduced transition probabilities should be given whenever possible. For mixed transitions, values for each multipole component should be given.

Note 1: When δ is consistent with zero or infinity, the reduced transition probability for only the dominant component is required. The limit for the other component is optional, but could be given in certain cases. Thus, BE2(W.u.)<1000 is not of much interest, but an unusually small value, say BE2(W.u.)$<10^{-3}$, might be significant.

Note 2: For transitions where the multipolarity has not been established but where all other needed data are available, it is recommended that values also be given when the probable γ character can be determined from the level scheme as ΔJ=1, Δπ =yes; ΔJ=2, Δπ =no, or ΔJ≥3, that is, cases where significant mixing is not expected. In such cases the multipolarity can be entered as [E1], [E2] or [E3], [M3] etc., respectively.

Note 3: When one or more of the relevant pieces of information needed to calculate reduced transition probabilities is missing, the calculation should be carried out if reasonable assumptions can be made that will fill in the gaps.

Example: If a level has one or more branches with small gamma fractions but unknown multipolarity, and if any reasonable multipolarity, say D or E2 would lead to the total branching also being relatively small, such branches should be estimated so that reduced transition probabilities for the strong transitions can be calculated.

2. For a transition whose total intensity is known only as an upper limit, then provided that this intensity limit is not the dominant branching mode for the level, it is recommended that the branching for this transition be treated as $\frac{1}{2}I_{\gamma+ce} \pm \frac{1}{2}I_{\gamma+ce}$ for the purpose of calculating the reduced transition probabilities for the other transitions.

3. When T½ itself is available only as an upper limit, it is recommended that the resulting lower limits on the reduced transition probabilities be given. When T½ is a lower limit, the resulting upper limits on the reduced transition probabilities are usually not very interesting, but see **1. Note 1:** above.

4. When a level T½ has been adopted from a measured BE2, the BE2W for the corresponding transition should be calculated directly from this measured BE2 rather than via the adopted T½. This procedure avoids the double counting of uncertainties. This procedure can also be followed in cases where BE2 is known but branchings and/or mixing ratios are not known so that T½ for the corresponding level cannot be calculated, The formula is given in **Appendix C**

5. In cases where Eγ is poorly known and α is significant, note that the factor $E\gamma^{2L+1}x(l+\alpha)$ appearing in the denominator of the formula for the reduced transition probabilities may exhibit a smaller range of values than the factors $E\gamma^{2L+1}$ and $(l+\alpha)$ taken separately. The correlation in Eγ and α should always be taken into account in calculating uncertainties for the reduced transition probabilities.

6. Reduced transition probabilities are not needed for mixed multipolarities where δ is not known; however, if an evaluator chooses to give them, they should be given as upper limits.

3.6. Electric monopole strength

RULER should be run to generate the absolute electric monopole strength, $\rho^2(E0)$. This quantity will appear in the output on a continuation record. See **section 1.11. E0 transitions, page 32** for a description of the input data needed for the $\rho^2(E0)$ calculation. A discussion, with formulae, is given in **Appendix D**.

> **Note:** Since $\rho^2(E0)$ generally lies in the range 10^{-1} to 10^{-3}, the output values from RULER are by convention quoted in units of 10^3 x $\rho^2(E0)$.

4. Analysis and Utility Programs

4.1. General

1. Evaluators should make use of the analysis and utility programs in preparing their mass chains. A list of all the programs is available at NNDC along with brief descriptions and is also given below for convenience. If the programs are kept on your computer, be sure to update them whenever NNDC issues a new version. As an alternative, the programs can be run via the NNDC website which ensures that the newest versions are being used.

> **Note:** Errors found in any of the analysis programs should be reported to NNDC.

.2. Be sure to check the output of all programs to be sure that the values produced are reasonable. This is especially important in cases where limits are involved, or where input values to a program overlap zero.

> **Example 1:** In a β^- decay dataset, $I_{\beta-}$ was entered as 6 ±9 and the logft program gave logft=6.6 ±0.7. This is clearly wrong and the program will be modified; however, it is recommended that in such a case the feeding should be entered as an upper limit. An input β^- intensity of <15 would have been correctly calculated by the logft program as logft > 6.2.
>
> **Example 2:** An M1+E2 transition was given with δ<0.1. For the particular case with given Eγ and T½ the output from RULER gave BM1W>0.14. There should clearly be an upper limit corresponding to δ=0 and for pure M1 one gets BM1W=0.15. The correct output should thus have been BM1W=0.145 ±0.005. This problem along with several others will be fixed in RULER, but at present this, and **Example 1** are illustrations of why the output from the programs should be looked at carefully.

3. It is mandatory that **FMTCK** be run and that all fatal errors be corrected before submitting the mass chain to NNDC.

4. For datasets with gammas, it is recommended that the excitation energies be obtained from the least-squares adjustment program, GTOL. Keep in mind that if the input energies are given without uncertainties, GTOL will add an uncertainty of 1 keV to each transition. See discussion in **section 1.4. Uncertainties, Comment 2, page 19.**

Analysis Programs

ALPHAD - Calculates α's, HF's and theoretical T½(α)'s.
Bricc - Interpolates Band-Raman internal conversion coefficients, internal electron-positron pair formation coefficients, and E0 form factors.
DELTA - Analyzes angular correlation data.
GABS - Calculates absolute Iγ's and normalizations.
GTOL - Determines level energies from a least-squares fit to Eγ's & calculates level feedings.
HSICC - Interpolates Hager-Seltzer and Dragoun internal conversion coefficients.
LOGFT - Calculates log ft, average beta energies, and capture fractions.
PANDORA - Physics check of ENSDF data sets. Aids with Adopted Gammas & XREF.
RadList - Calculates atomic & nuclear radiations. Checks energy balance.
RULER - Calculates reduced transition probabilities.

Utility Programs

ADDGAM - Adds gammas to adopted dataset.
AveTools - The program AveTools combines three different statistical methods to calculate

averages of experimental data with uncertainties.

Visual Averaging Library - The program V.AvLib offers eight methods of averaging values and testing for outliers.

ENSDAT- Produces Nuclear Data Sheets style tables and drawings

ComTrans - Converts the text comments of an ENSDF dataset to a "rich text format".

FMTCK - ENSDF format checking.

TREND - Tabular display of ENSDF data.

XLS2ENS - Convert Datasets in Excel Spreadsheets to ENSDF Format

Appendices

A. Calculation of Radius Parameters for Alpha decay of Odd-A and Odd-Odd Nuclides*

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As discussed in 1998Ak04, the nuclear radius parameter, r_0, for an even-even nuclide is determined by defining the calculated transition probability for an alpha decay from the ground state of the parent to the ground state of the daughter to be equal to the experimental transition rate, that is, HF=1.0 for such a transition. The radius parameter calculated by our alpha hindrance factor program is that of the daughter, that is, the effective radius for the alpha decay is given by $R=r_0A^{1/3}$, where A is that of the daughter nuclide. The r_0 parameters for the even-even nuclides are summarized in 1998Ak04. Note that in Table I of this reference, the first two columns give the Z and N of the alpha-decaying parent, while the values of r_0 correspond to the Z-2, N-2 daughter.

Radius parameters for the odd-A and odd-odd nuclides, needed as input for the alpha hindrance program, are obtained from the parameters for the adjacent even-even nuclides.

Figure 1 of 1998Ak04 shows r_0 as a function of N for the even-Z nuclides. It is assumed that the parameters for the odd-Z nuclides lie midway between those for the adjacent Z-1 and Z+1 nuclides. Thus, for example, the r_0 parameters for the Z=85 Astatine daughter nuclides trace out a curve midway between the curves for the Z=84 Polonium and Z=86 Radon nuclides.

It is also assumed that for the even-Z, odd-N case, the parameters for the odd-N isotopes lie midway between the values for the Z, N-1 and Z, N+1 even-even nuclides.

Note that in what follows, all averages are unweighted. A weighted average is not appropriate. Keep in mind that one is not trying to get a best value for two measurements of the same quantity, but rather a value midway between parameters for adjacent nuclides.

1. Odd-Z, Even-N Nuclides

The r_0 parameter for an odd-Z, even-N nuclide is obtained as an unweighted average of the values for the nuclides with Z-1, N and Z+1, N. The uncertainty is obtained as an unweighted average of the upper (or lower) limits of these adjacent nuclide values.

* S. Singh and B Singh have developed the RadD program to deduce the r_0 parameters. See IAEA report INDC(NDS)-0665, September 2014.

2. Even-Z, Odd-N Nuclides

The r_0 parameter for an even-Z, odd-N nuclide is obtained as an unweighted average of the values for the nuclides with Z, N-1 and Z, N+1. The uncertainty is obtained as an unweighted average of the upper (or lower) limits of these adjacent nuclide values.

3. Odd-Z, Odd-N Nuclides

For an odd-odd nuclide, Z, N one needs r_0 parameters for the four nuclides with Z±1, N±1. In step one, r_0 values for Z, N-1 and for Z, N+1 are obtained as described in 1. above. The value for Z, N is then obtained as an average of the two values from step one. Alternatively, one can average the values for the Z-1, N and Z+1, N nuclides, each of which is obtained as described in 2. above. The two approaches give the same value of r_0.

Example

The following example illustrates the above procedures for the alpha decay of the odd-Z Francium nuclides with Z=87 and N=118, 119, and 120, decaying to the daughter Astatine nuclides with Z=85 and N=116, 117, and 118, respectively. One needs the daughter radius parameters from the four parent alpha decays with Z=86, 88 and N=118, 120. From 1998Ak04 these values are

^{204}Rn (Z=86, N=118) ➜ $r_0(^{200}\text{Po}) = 1.504$ *3*

^{206}Rn(Z=86, N=120) ➜ $r_0(^{202}\text{Po}) = 1.492$ *7*

^{206}Ra(Z=88, N=118) ➜ $r_0(^{202}\text{Rn}) = 1.527$ *8*

^{208}Ra(Z=88, N=120) ➜ $r_0(^{204}\text{Rn}) = 1.495$ *14*

These four points form the four corners of a grid, as given by the values in bold in the following table. Z and N here correspond to the daughter.

	N=116	N=117	**N=118**
Z=84	**^{200}Po r_0 =1.504** ***3***	^{201}Po r_0=1.498 *5*	**^{202}Po r_0=1.492** *7*
Z=85	^{201}At r_0=1.5155 *55*	^{202}At r_0=1.5045 *80*	^{203}At r_0=1.4935 *105*
Z=86	**^{202}Rn r_0=1. 527** ***8***	^{203}Rn r_0=1.511 *11*	**^{204}Rn r_0=1.495** *14*

The value for ^{201}Po is obtained by taking an unweighted average of the parameters for ^{200}Po and ^{202}Po, and that for ^{201}At comes from an unweighted average of the values for ^{200}Po and ^{202}Rn, and so on. The uncertainties come from an unweighted average of the corresponding maximum (or minimum) values. Thus, for ^{201}At, the average of 1.504 and 1.527 gives 1.5155, and the average of the upper limits, 1.507 and 1.535, gives 1.5210 which corresponds to an uncertainty of +55 on the value 1.5155. The lower limits of course give the same uncertainty.

The value for the odd-odd ^{202}At can be obtained by averaging the values for ^{201}Po and ^{203}Rn, or the values for ^{201}At and ^{203}At. The two approaches give the same value of 1.5045 *80*.

The Astatine values should of course be rounded off for publication following our usual policy; however, see the second paragraph in the following section.

Regional r_0 Systematics

It is strongly recommended to evaluators working in mass regions with alpha decay that they plot the r_0 parameters for the nuclides in and around their mass regions. These plots can be of use in estimating parameters when critical even-even data are missing. For example, suppose that in the above example the r_0 parameter for ^{202}Po was not known experimentally. It might still be possible to obtain a reasonable estimate of the value by extrapolation from the known lower-N Po values. Similarly, one might be able to estimate a value for ^{202}At or ^{203}At by extrapolation from values for lower-N At nuclides. Even an approximate value might be sufficient to establish a particular alpha branch as being favored. Of course such extrapolations are worth doing only if the r_0 plots are fairly regular, and the regularity, or lack thereof, is something that is useful to know in one's mass region.

In keeping these r_0 systematics, it is recommended that more digits be retained than would correspond to our usual policy. This will avoid possible cumulative roundoff problems. In the above example, r_0(^{202}At) could be retained in an evaluator's internal file of parameters as 1.5045 80, with the roundoff to 1.504 8 being done at the mass chain stage.

B. Sign Conventions for Mixing Ratios from Angular Correlations and Angular Distributions in Electromagnetic Transitions

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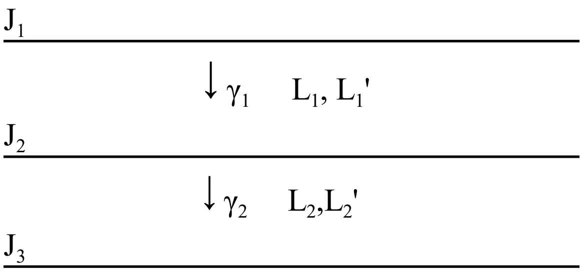
August, 1987
Revised with minor changes in the text, and a correction to the LHS of the present equation (8), original equation (13).
September, 2016

Since the multipole components in a gamma transition of mixed character mix coherently, an angular distribution or angular correlation measurement can determine the phases, that is, the relative signs of these components. The following discussion is restricted to the case where two components with multipole orders L and L' =L+1 compete. Since the relative phase of these two components is invariant with respect to any arbitrary phase convention for the wave functions or the transition operators, it is a physically significant quantity.

The two most widely used phase conventions are those proposed by Steffen [1] and by Rose and Brink [2] but there are many more that have appeared in the literature. Table I gives a summary of the relation between the phase conventions used by these authors relative to that of Steffen, whose convention is adopted in the following material.

General Formalism

Consider the following two-gamma cascade connecting levels with spins J_1, J_2, and J_3 with each transition consisting of just two multipoles, L and L' = L+1.



Expressions for the directional correlation of γ_1 and γ_2 can be reduced to the form

$$W(\theta) = \Sigma B_k(\gamma_1) A_k(\gamma_2) P_k(\cos\theta) \qquad (1)$$

with the sum including only even k.

where the B_k are orientation parameters and the A_k are directional distribution coefficients. The parameters B_k are characteristic of the (axially symmetric) orientation of the intermediate state, J_2 in our example, while the coefficients A_k characterize the directional distribution of γ_2 with respect to the orientation axis of J_2. The following discussion describes the case where the intermediate state is oriented by the observation of the incoming radiation γ_1 from the state J_1.

> **Note:** It is pointed out in reference [20] that the restriction to even k in (1) holds in cases where the states involved have pure parity and the radiations are parity conserving, e.g. electromagnetic radiation, conversion electrons, and alpha particles. $A_k = 0$ for odd k for such cases. If the circular polarization of the gammas is observed, if the nuclear states have parity admixtures, if β's are observed, then, if the intermediate state is polarized and not just aligned, odd-k terms can contribute.

For a given transition, γ, we denote the relative amplitude of the multipole orders L' + 1 and L by δ(γ) as defined in (4) and (5) below. The sign of δ is dependent on several factors:

a) The form in which eq. (1) is expanded
b) The choice of matrix elements, emission or absorption
c) The form of the Wigner-Eckhart theorem used to define the reduced matrix elements
d) The form of the electromagnetic multipole transition operators

For the extraction of δ from the analysis of an angular correlation or angular distribution experiment only a) is relevant. The B_k and A_k can be expanded in terms of tabulated constants which depend only on the L's and J's and that expansion fixes the sign of δ. If, however, an attempt is made to calculate the mixing ratios on the basis of some model and to compare them with values deduced from experiment, then b), c), and d) must be taken into account.

Following the work of Steffen [1], for the case of an unpolarized, unaligned initial state J_1, the B_k and A_k can be expressed as

$$B_k(\gamma_1) = \frac{F_k(L_1 L_1 J_1 J_2) + 2\delta(\gamma_1)(-1)^{L-L'+k} F_k(L_1 L_1' J_1 J_2) + \delta^2(\gamma_1) F_k(L_1' L_1' J_1 J_2)}{1+\delta^2(\gamma_1)} \tag{2}$$

$$A_k(\gamma_2) = \frac{F_k(L_2 L_2 J_3 J_2) + 2\delta(\gamma_2) F_k(L_2 L_2' J_3 J_2) + \delta^2(\gamma_2) F_k(L_2' L_2' J_3 J_2)}{1+\delta^2(\gamma_2)} \tag{3}$$

where

$$\delta(\gamma_1) = \langle J_2 \| \pi_1' L_1' \| J_1 \rangle / \langle J_2 \| \pi_1 L_1 \| J_1 \rangle \tag{4}$$

$$\delta(\gamma_2) = \langle J_3 \| \pi_2' L_2' \| J_2 \rangle / \langle J_3 \| \pi_2 L_2 \| J_2 \rangle \tag{5}$$

The coefficients F_k are defined and tabulated by Frauenfelder and Steffen [3]. They are also tabulated in references [4, 5, 6]. Steffen uses emission matrix elements and the initial state always appears on the right. Equations (1), (2), and (3) define a unique sign for the mixing ratios. Note that the coefficients F_k are defined such that

$$F_0(LL'J_2J_3) = \delta_{LL'}$$

where $\delta_{LL'}$ is the Kronecker delta, and thus

$$B_0(\gamma) = A_0(\gamma) = 1$$

As an example, for the cascade $J_1=4$, $J_2=2$, $J_3=0$, the tables in reference [3] give

$$F_2(2242) = -0.1707,\ F_2(2202) = -0.5976$$

The definition of operators and reduced matrix elements used by Rose and Brink [2] are such that

$$\delta_{Rose-Brink} = (-1)^{L-L'}\delta_{Steffen}$$

Note that Rose and Brink use absorption matrix elements and the initial state always appears on the left. As a consequence, in the work of these authors the coefficients F_k are replaced by coefficients R_k, tabulated in [2] and defined so that

$$R_k(LL'J_iJ_f) = (-1)^{L-L'+k}F_k(LL'J_fJ_i)$$

but otherwise the forms of equations (2) and (3) remain unchanged. Both Rose - Brink and Steffen give a thorough discussion of the factors a) through d) mentioned above, so that calculations of δ can be carried through consistently in either formalism.

Another phase convention that one still encounters occasionally, although mainly in older papers, is that of Biedenharn and Rose [4]. These authors adopt the convention of always writing the intermediated state, J_2 in our example, on the right. This convention leads to an additional phase factor of $(-1)^{L-L'+k}$ in the second term of equation (2), thus

$$\delta_{Biedenharn-Rose}(\gamma_1) = -\delta_{Steffen}(\gamma_1)$$

Note that in the Biedenharn and Rose formalism, the sign of the mixing ratio depends on whether a given transition appears first or second in a cascade.

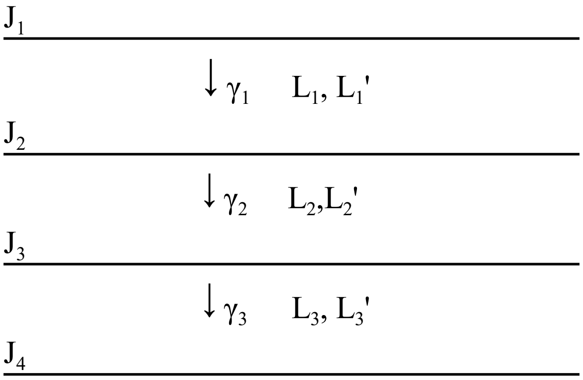
Table I gives a summary of the phase conventions one might find in the literature. They are all given here relative to the convention of Steffen, where the sign of the mixing ratio for both the first and second transition is arbitrarily set to be "+". Note that in the formalism of references [9], [10], and [16] the sign depends on whether or not the transition involves a parity change.

Special Cases

Some special cases the evaluators might come across are described in the following sections.

1. Intermediate Unobserved Radiation

Consider the cascade J_1 (γ_1) J_2 (γ_2) J_3 (γ_3) J_4 where the directional distribution of γ_3 is measured relative to the direction of γ_1 and radiation γ_2 is unobserved.



This case can be treated using equation (1) with the addition of a multiplicative de-orientation factor $U_k(LJ_2J_3)$ defined by

$$U_k(L_2J_2J_3) = \frac{(-1)^{J_2+J_3}[(2J_2+1)(2J_3+1)]^{1/2}\left[(-1)^{L_2}\begin{Bmatrix} J_2J_2k \\ J_3J_3L_2 \end{Bmatrix} + (-1)^{L_2'}\Delta^2(\gamma_2)\begin{Bmatrix} J_2J_2k \\ J_3J_3L_2' \end{Bmatrix}\right]}{1+\Delta^2(\gamma_2)} \qquad (6)$$

$$= \frac{U_k(L_2J_2J_3)+\Delta^2(\gamma_2)U_k(L_2'J_2J_3)}{1+\Delta^2(\gamma_2)}$$

where

$$\Delta^2(\gamma_2) = \frac{1+\alpha(L_2'\pi')}{1+\alpha(L_2\pi)}\delta^2(\gamma_2)$$

α(L') and α(L) being the total conversion coefficients for the L_2+1 and L_2 components, respectively of γ_2, and $\delta(\gamma_2)$ is defined by equation (5). The { } are 6-J symbols.

The $U_k(LJJ')$ are called de-orientation coefficients since they take account of the decrease in alignment resulting from the unobserved transition. They are tabulated in [2], [16], and [17]. Equation (6) is from Anicin et al. [21]. Explicit expressions given in all other sources with which

I am familiar are incorrect in that they contain δ^2 in place of Δ^2. It should be noted that Δ is parity dependent through the factor $(1 + \alpha(L_2'\pi')) / (1 + \alpha(L_{2\pi}))$. For α's $\ll 1$ this factor reduces to unity, and $\Delta \rightarrow \delta$, but for large α's, this factor can distinguish, for example, M1+E2 from E1+M2.

Note that equation (6) contains no interference term since the radiation it represents is not observed. Note also that $U_0(JJ) = 1$

The angular distribution for the general case of n radiations with the correlation between the first and last transition being measured is given, by extension, by

$$W(\theta) = \sum_{k=even} B_k(\gamma_1)U_k(\gamma_2)...U_k(\gamma_{n-1})A_k(\gamma_n)P_k(\cos\theta) \tag{7}$$

2. Correlations involving Internal Conversion electrons

If internal conversion electrons, rather than gammas, are observed for either of the transitions in the cascade described by equation (1), then the factors F_k appearing in equations (2) and (3) should be replaced by the factors $b_k(ce_i\Lambda\Lambda')F_k$, where the $b_k(ce_i\Lambda\Lambda')$ are particle parameters for conversion in the i shell for Λ, Λ' = M1, E2, or E1, M2, etc., tabulated in [22], and the mixing ratio δ should be replaced by

$$\delta(ce_i) = \left[\frac{\alpha_i(L'\pi')}{\alpha_i(L\pi)}\right]^{1/2} \delta(\gamma)$$

For a $(\gamma)(ce_i)(\theta)$ experiment where the second transition is M1+E2, for example, equation (3) becomes

$$\begin{gathered} A_k(ce_i)[1+\delta^2(ce_i)] = \\ F_k(11J_3J_2)b_K(ce_iM1,M1) + 2\delta(ce_i)F_k(12J_3J_2)b_k(ce_iM1,E2) + \\ \delta^2(ce_i)F_k(22J_3J_2)b_k(ce_iE2,E2) \end{gathered} \tag{8}$$

where the $1+\delta^2$ term has been transposed to the LHS.

3. Resonance Fluorescence

The angular distribution in a (γ,γ') experiment, where the exciting and deexciting transitions are the same, follows from equations (1), (2), and (3) with the further observation that the reduced matrix elements defined in equations (4) and (5) have, for this case, the property that

$$\delta(\gamma_1) = -\delta(\gamma_2)$$

For the sequence $J_3(L_2L_2+1)J_2(L_2L_2+1)J_3$ one then has

$$W(\theta) = \sum_{k=even} A_k^{\,2}(\gamma_2)P_k(\cos\theta) \tag{9}$$

with $A_k(\gamma_2)$ given by equation (3) and $\delta(\gamma_2)$ given by equation (5). The more general case of (γ,γ') is treated like γγ(θ) with the ground state as the initial state, J_1.

4. Coulomb Excitation

The angular distribution of gammas in a Coulomb excitation experiment takes the form of equation (1) with

$$B_k(\gamma_1) = b_k(\S)F_k(22J_1J_2) \tag{10}$$

where J_1 is the target spin, J_2 the spin of the Coulomb-excited state, J_3 is the spin of the final level following the gamma emission, and $b_k(\S)$ is a particle parameter which depends on the excitation process through the parameter §. These particle parameters are tabulated in [5].

5. Alpha Decay

The form for the αγ(θ) correlation is similar to that described above for ceγ(θ). The factors F_k in the expression for B_k are multiplied by the particle parameters for alpha decay, b_k, defined by

$$b_k(LL") = b_k(L"L) = \cos(\eta_L - \eta_{L"})\frac{2[L(L+1)L"(L"+1)]^{1/2}}{L(L+1)+L"(L"+1)-k(k+1)} \tag{11}$$

where L" = L+2, L+4 etc., and δ(γ) is replaced by

$$\delta(\alpha) = < J_2 \| H_{L"}(\alpha) \| J_1 > / < J_2 \| H_L(\alpha) \| J_1 >$$

For alpha decay in which a single L value dominates, B_k takes the same form as equation (10), namely

$$B_k(\alpha) = b_k(LL)F_k(LLJ_1J_2)$$

For mixed-L transitions, since L+L" = even, the phase factor in the second term of equation (2) becomes +1, and equations (2) and (3) have exactly the same form. The phase angle appearing in equation (11) contains the Coulomb phase shifts and depends on the target nuclide and the alpha particle energy. It enters only in the mixed L, L" term of equations (2). See references [3] and [23]* for a definition of this phase term.

* Note that in equation (123) of [23], the cross term should be multiplied by a factor of 2.

The difference in phase angles for L and L" are usually small (see reference [24] and references contained therein) so that the cos term is close to +1 or -1. We adopt $\cos(\eta_L - \eta_{L''}) = +1$ which, along with the forms of equations (12) and (13) below, defines the phase of $\delta(\alpha)$.

For the case where only the two lowest L values contribute significantly, equation (2) becomes

$$\begin{gathered} B_k(\alpha)[1+\delta^2(\alpha)] = \\ b_k(LL)F_k(LLJ_1J_2) + 2\delta(\alpha)b_k(LL'')F_k(LL''J_1J_2) + \delta^2 b_k(L''L'')F_k(L''L''J_1J_2) \end{gathered} \tag{12}$$

where the $(1+\delta^2)$ term has been transposed to the LHS of the equation. For the case of L=0, the particle parameter cannot be defined in terms of the functions F_k for the gammas. For L=0+2, using equation (7.10) of Steffen [1b], or equation (107) of [1c], equation (12) for k=2 becomes

$$B_2(\alpha) = \frac{2\delta_{J_1J_2}\delta(\alpha)\cos(\eta_2 - \eta_0) + \delta^2(\alpha)b_2(22)F_2(22J_2J_1)}{\delta_{J_1J_2} + \delta^2(\alpha)} \tag{13}$$

where $\delta_{J_1J_2}$ is the Kronecker delta. For a pure L=0 alpha transition, the angular distribution is isotropic.

6. Other Cases

When the intermediate state, J_2 in Fig. 1, is oriented by low-temperature techniques or by nuclear reactions etc., the angular distribution can be described by equation (7), with the B_k now treated as alignment parameters which may be determined experimentally, estimated empirically, or calculated on the basis of a specific model [17]. See for example references [1], [2], and the tabulations and references quoted in references [17] and [18].

Table I

Sign of δ normalized to convention of Steffen		**Reference**
γ_1	**γ_2**	
+ (nomalization)	+ (normalization)	Steffen [1], [14], [15]
+	+	Frauenfelder-Steffen [3]
+	+	Poletti-Start [13]
+	+	Taylor et al. [8]
+	+	Yamazaki [17]
+	+	Ferguson [19]
-	+	Biedenharn-Rose [4]
-	+	Ferguson-Rutledge (1957) [7]
Pure E2 assumed	+	Alder et al. [5]
+ (Δπ=yes) - (Δπ=no)	+ (Δπ=yes) - (Δπ=no)	Litherland-Ferguson [10] Poletti-Warburton [16] Ferguson-Rutledge (1962) [9]
-	-	Rose-Brink [2]
-	-	Smith [11]
-	-	Harris et al. [12]
-	-	Watson-Harris [18]

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C. Reduced Gamma-Ray Matrix Elements, Transition Probabilities, and Weisskopf Single-Particle Estimates

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September, 1982
Revised with minor changes in the text, and with some δ factors included explicitly. Some additional expressions have been added
September, 2016

The formulas given below for Weisskopf estimates BλW are the basis for the calculations in the program RULER.

For an electromagnetic transition of energy Eγ, the relationships among the reduced matrix elements, $B(\sigma L)$, and the partial gamma-ray half-lives, $T_{1/2}^{\gamma}$, are

$$T_{1/2}^{\gamma}(EL)xB(EL)\downarrow=\frac{(\ln 2)L[(2L+1)!!]^2(h/2\pi)}{8\pi(L+1)e^2b^L}\left(\frac{hc/2\pi}{E_\gamma}\right)^{2L+1} \quad (1)$$

and

$$T_{1/2}^{\gamma}(ML)xB(ML)\downarrow=\frac{(\ln 2)L[(2L+1)!!]^2 h/2\pi}{8\pi(L+1)\mu_N^2b^{L-1}}\left(\frac{hc/2\pi}{E_\gamma}\right)^{2L+1} \quad (2)$$

The Weisskopf single-particle estimates for the $B(\sigma L)$ are

$$B(s.p.)(EL)\downarrow=\frac{1}{4\pi b^L}\left(\frac{3}{3+L}\right)^2 R^{2L} \quad (3)$$

$$B(s.p.)(ML)\downarrow=\frac{10}{\pi b^{L-1}}\left(\frac{3}{3+L}\right)^2 R^{2L-2} \quad (4)$$

so that

$$T_{1/2}^{\gamma}(s.p.)(EL)=\frac{(\ln 2)L[(2L+1)!!]^2 h/2\pi}{2(L+1)e^2R^{2L}}\left(\frac{(3+L}{3}\right)^2\left(\frac{hc/2\pi}{E\gamma}\right)^{2L+1} \quad (5)$$

and

$$T_{1/2}^{\gamma}(s.p.)(ML)=\frac{(\ln 2)L[(2L+1)!!]^2 h/2\pi}{80(L+1)\mu_N^2R^{2L-2}}\left(\frac{3+L}{3}\right)^2\left(\frac{hc/2\pi}{E\gamma}\right)^{2L+1} \quad (6)$$

The relationship between a measured $B(\sigma L)\uparrow$ to a level with spin J_f and half-life $T_{1/2}(J_f)$ from a level with spin J_i connected by a transition γ_k with mixing ratio δ_k is given by Eq. (1) or (2) with

$$T_{1/2}(J_f) = T_{1/2}^{\gamma_k}(EL)\varepsilon(\gamma_k)\frac{\delta_k^2}{1+\delta_k^2} = T_{1/2}^{\gamma_k}(ML)\varepsilon(\gamma_k)\frac{1}{1+\delta_k^2} \tag{7}$$

and

$$B(\sigma L)\uparrow = \frac{(2J_f+1)}{(2J_i+1)}B(\sigma L)\downarrow$$

where $\varepsilon(\gamma_k)$ is the fraction of the decays of level J_f that proceed by the observed mode γ_k and is defined by

$$\varepsilon(\gamma_k) = \frac{\lambda_k^\gamma}{\sum_i (1+\alpha_i)\lambda_i^\gamma} = \frac{BR(\gamma_k)}{(1+\alpha_k)}$$

In the above equations, $b = 10^{-24}cm^2$, $R = R_0 A^{1/3}x10^{-13}cm$, and $B(EL)$ and $B(ML)$ are in units of e^2b^L and $\mu_N^2 b^{L-1}$, respectively.

Explicit expressions for the above equations are given below for L=1 to 5. E_γ is in keV, $T_{1/2}$ is in seconds, Γ is in eV, and $(W.u.)$ stands for Weisskopf units. The mixing ratio factors are included explicitly for L=1, 2, and 3. It is unlikely that they would be needed for higher multipoles but they could be taken into account if necessary.

The Weisskopf estimates for E1 and M1 transitions are expressed in terms of both $T_{1/2}$ and width, and for E2 transitions in terms of both $T_{1/2}$ and $B(E2)$. See the Guidelines for details as to when and why the alternate expressions should be used.

The constants adopted are as follows:

$$hc/2\pi = 1.9733x10^{-8}keV-cm$$
$$h/2\pi = 0.6584x10^{-18}keV-s$$
$$e^2 = 1.43998x10^{-10}keV-cm$$
$$\mu_N^2 = 1.59234x10^{-38}keV-cm^3$$
$$R_0 = 1.2$$

E1 Transitions

$$T^{\gamma}_{1/2}(E1)xB(E1)\downarrow = \frac{4.360x10^{-9}}{E^3_\gamma}$$

$$B(s.p.)(E1)\downarrow = 6.446x10^{-4}\,A^{2/3}(e^2x10^{-24}cm^2)$$

$$T^{\gamma}_{1/2}(s.p.)(E1) = \frac{6.764x10^{-6}}{E^3_\gamma A^{2/3}}$$

$$B(E1)(W.u.) = \frac{6.764x10^{-6}BR}{E^3_\gamma A^{2/3}T_{1/2}(1+\alpha)}\left(\frac{1}{1+\delta^2}\right) = \frac{1.4827x10^{10}BR}{E^3_\gamma A^{2/3}(1+\alpha)}\left(\frac{1}{1+\delta^2}\right)\Gamma$$

$$T_{1/2}(J_f)xB(E1)\uparrow = \frac{4.360x10^{-9}BR}{E^3_\gamma(1+\alpha)}\left(\frac{2J_f+1}{2J_i+1}\right)\left(\frac{1}{1+\delta^2}\right)$$

E2 Transitions

$$T^{\gamma}_{1/2}(E2)xBE2\downarrow = \frac{56.59}{E^5_\gamma}$$

$$B(s.p.)(E2)\downarrow = 5.940x10^{-6}\,A^{4/3}\,(e^2x10^{-48}cm^4)$$

$$T^{\gamma}_{1/2}(s.p.)(E2) = \frac{9.527x10^{6}}{E^5_\gamma A^{4/3}}$$

$$B(E2)(W.u.) = \frac{9.527x10^{6}BR}{E^5_\gamma A^{4/3}T_{1/2}(1+\alpha)}\left(\frac{\delta^2}{1+\delta^2}\right)^{*} = \frac{1.6835x10^{5}}{A^{4/3}}\left(\frac{2J_i+1)}{2J_f+1}\right)B(E2)\uparrow$$

$$T_{1/2}(J_f)xB(E2)\uparrow = \frac{56.59BR}{E^5_\gamma(1+\alpha)}\left(\frac{\delta^2}{1+\delta^2}\right)^{*}\left(\frac{2J_f+1)}{2J_i+1}\right)$$

$*$ Where $\delta = \left|\frac{E2}{M1}\right|$. For $\delta = \left|\frac{M3}{E2}\right|$, this factor should be replaced by $\frac{1}{1+\delta^2}$

E3 Transitions

$$T_{1/2}^{\gamma}(E3)xB(E3)\downarrow=\frac{1.215x10^{12}}{E_{\gamma}^{7}}$$

$$B(s.p.)(E3)\downarrow=5.940x10^{-8}A^{2}\,(e^{2}x10^{-72}cm^{6})$$

$$T_{1/2}^{\gamma}(s.p.)(E3)=\frac{2.045x10^{19}}{E_{\gamma}^{7}A^{2}}$$

$$B(E3)(W.u.)=\frac{2.045x10^{19}BR}{E_{\gamma}^{7}A^{2}T_{1/2}(1+\alpha)}\left(\frac{1}{1+\delta^{2}}\right)^{*}$$

$$T_{1/2}(J_{f})xB(E3)\uparrow=\frac{1.215x10^{12}BR}{E_{\gamma}^{7}(1+\alpha)}\left(\frac{1}{1+\delta^{2}}\right)^{*}\left(\frac{2J_{f}+1}{2J_{i}+1}\right)$$

$*$ Where $\delta=\left|\frac{M4}{E3}\right|$. For $\delta=\left|\frac{E3}{M2}\right|$, this factor should be replaced by $\frac{\delta^{2}}{1+\delta^{2}}$.

E4 Transitions

$$T_{1/2}^{\gamma}(E4)xB(E4)\downarrow=\frac{4.087x10^{22}}{E_{\gamma}^{9}}$$

$$B(s.p.)(E4)\downarrow=6.285x10^{-10}A^{8/3}\;\;(e^{2}x10^{-96}cm^{8}$$

$$T_{1/2}^{\gamma}(s.p.)(E4)=\frac{6.503x10^{31}}{E_{\gamma}^{9}A^{8/3}}$$

$$B(E4)(W.u.)=\frac{6.503x10^{31}BR}{E_{\gamma}^{9}A^{8/3}T_{1/2}(1+\alpha)}$$

$$T_{1/2}(J_{f})xB(E4)\uparrow=\frac{4.087x10^{22}BR}{E_{\gamma}^{9}(1+\alpha)}\left(\frac{2J_{f}+1}{2J_{i}+1}\right)$$

E5 Transitions

$$T_{1/2}^{\gamma}(E5)xB(E5)\downarrow = \frac{2.006x10^{33}}{E_{\gamma}^{11}}$$

$$B(s.p.)(E5)\downarrow = 6.929x10^{-12}A^{10/3}\ (e^2x10^{-120}cm^{10})$$

$$T_{1/2}(s.p.)(E5) = \frac{2.895x10^{44}}{E_{\gamma}^{11}A^{10/3}}$$

$$B(E5)(W.u.) = \frac{2.895x10^{44}BR}{E_{\gamma}^{11}A^{10/3}T_{1/2}(1+\alpha)}$$

$$T_{1/2}(J_f)xB(E5)\uparrow = \frac{2.006x10^{33}BR}{E_{\gamma}^{11}(1+\alpha)}\left(\frac{2J_f+1}{2J_i+1}\right)$$

Expressions for ML transitions can be expressed in terms of the EL formulae as follows:

$$T_{1/2}^{\gamma}(ML)/T_{1/2}^{\gamma}(EL) = 9.043x10^3 B(EL)/B(ML)$$

$$B(s.p.)(ML)/B(s.p.)(EL) = 2.778x10^3 A^{-2/3}$$

$$T_{1/2}^{\gamma}(s.p.)(ML)/T_{1/2}^{\gamma}(s.p.)(EL) = 3.256A^{2/3}$$

$$B(ML)(W.u.)/B(EL)(W.u.) = 3.256A^{2/3}$$

Explicit expressions are given below.

M1 Transitions

$$T_{1/2}^{\gamma}(M1)xB(M1)\downarrow = \frac{3.943x10^{-5}}{E_{\gamma}^{3}}$$

$$B(s.p.)(M1)\downarrow = 1.791\ (\mu_N^2)$$

$$T_{1/2}^{\gamma}(s.p.)(M1) = \frac{2.202x10^{-5}}{E_{\gamma}^{3}}$$

$$B(M1)(W.u.) = \frac{2.202x10^{-5}BR}{E_{\gamma}^{3}T_{1/2}(1+\alpha)}\left(\frac{1}{1+\delta^2}\right) = \frac{4.827x10^{10}\Gamma_0}{E_{\gamma}^{3}(1+\alpha)}\left(\frac{1}{1+\delta^2}\right)$$

$$T_{1/2}(J_f)xB(M1)\uparrow = \frac{3.943x10^{-5}BR}{E_{\gamma}^{3}(1+\alpha)}\left(\frac{1}{1+\delta^2}\right)\left(\frac{2J_f+1)}{2J_i+1}\right)$$

M2 Transitions

$$T_{1/2}^{\gamma}(M2)B(M2)\downarrow = \frac{5.117x10^{5}}{E_{\gamma}^{5}}$$

$$B(s.p.)(M2)\downarrow = 1.650x10^{-2}A^{2/3}\ (\mu_N^2 x10^{-24}cm^2)$$

$$T_{1/2}^{\gamma}(s.p.)(M2) = \frac{3.102x10^{7}}{E_{\gamma}^{5}A^{2/3}}$$

$$B(M2)(W.u.) = \frac{3.102x10^{7}BR}{E_{\gamma}^{5}A^{2/3}T_{1/2}(1+\alpha)}\left(\frac{\delta^2}{1+\delta^2}\right)^{*}$$

$$T_{1/2}(J_f)xB(M2)\uparrow = \frac{5.117x10^{5}BR}{E_{\gamma}^{5}(1+\alpha)}\left(\frac{\delta^2}{1+\delta^2}\right)^{*}\left(\frac{2J_f+1)}{2J_i+1}\right)$$

* Where $\delta = \left|\frac{M2}{E1}\right|$. For $\delta = \left|\frac{E3}{M2}\right|$, this factor should be replaced by $\frac{1}{1+\delta^2}$

M3 Transitions

$$T_{1/2}^{\gamma}(M3)xB(M3)\downarrow=\frac{1.099x10^{16}}{E_{\gamma}^{7}}$$

$$B(s.p.)(M3)\downarrow=1.650x10^{-4}\,A^{4/3}\ \ (\mu_N^2 x10^{-48}cm^4$$

$$T_{1/2}^{\gamma}(s.p.)(M3)=\frac{6.659x10^{19}}{E_{\gamma}^{7}A^{4/3}}$$

$$B(M3)(W.u.)=\frac{6.659x10^{19}\,BR}{E_{\gamma}^{7}A^{4/3}T_{1/2}(1+\alpha)}\left(\frac{\delta^2}{1+\delta^2}\right)^{*}$$

$$T_{1/2}(J_f)xB(M3)\uparrow=\frac{1.099x10^{16}\,BR}{E_{\gamma}^{7}(1+\alpha)}\left(\frac{\delta^2}{1+\delta^2}\right)^{*}\left(\frac{2J_f+1}{2J_i+1}\right)$$

* Where $\delta=\left|\frac{M3}{E2}\right|$. For $\delta=\left|\frac{E4}{M3}\right|$, this factor should be replaced by $\frac{1}{1+\delta^2}$

M4 Transitions

$$T_{1/2}^{\gamma}(M4)xB(M4)\downarrow=\frac{3.696x10^{26}}{E_{\gamma}^{9}}$$

$$B(s.p.)(M4)\downarrow=1.746x10^{-6}\,A^{2}\ \left(\mu_N^2 x10^{-72}cm^6\right)$$

$$T_{1/2}^{\gamma}(s.p.)(M4)=\frac{2.117x10^{32}}{E_{\gamma}^{9}A^{2}}$$

$$B(M4)(W.u.)=\frac{2.117x10^{32}\,BR}{E_{\gamma}^{9}A^{2}T_{1/2}(1+\alpha)}$$

$$T_{1/2}(J_f)B(M4)\uparrow=\frac{3.696x10^{26}\,BR}{E_{\gamma}^{9}(1+\alpha)}\left(\frac{2J_f+1}{2J_i+1}\right)$$

M5 Transitions

$$T_{1/2}^{\gamma}(M5)xB(M5)\downarrow = \frac{1.814x10^{37}}{E_{\gamma}^{11}}$$

$$B(s.p.)(M5)\downarrow = 1.925x10^{-8}A^{8/3}\ (\mu_N^2 x10^{-96}cm^8)$$

$$T_{1/2}^{\gamma}(s.p.)(M5) = \frac{9.426x10^{44}}{E_{\gamma}^{11}A^{8/3}}$$

$$B(M5)(W.u.) = \frac{9.426x10^{44}BR}{E_{\gamma}^{11}A^{8/3}T_{1/2}(1+\alpha)}$$

$$T_{1/2}(J_f)xB(M5)\uparrow = \frac{1.814x10^{37}BR}{E_{\gamma}^{11}(1+\alpha)}\left(\frac{2J_f+1}{2J_i+1}\right)$$

Miscellaneous Relations

Recoil correction for gamma transitions

$$E_{\gamma}(recoil)(keV) = \frac{5.3677x10^{-7}E_{\gamma}^{2}(keV)}{A}$$

Relationship between half-life and total width

$$\mathit{\Gamma} xT_{1/2} = 0.45623\ \text{x}\ 10^{-15}eV - s$$

D. Monopole Transition Strength

Excerpted by M. J. Martin from Ref [1]
September, 2016

The electric monopole (E0) transition probability is defined by

$$W(E0) = \frac{\ln 2}{\mathrm{T}_{1/2}(E0)} = W_{ic}(E0) + W_{\pi}(E0)$$

where $T_{½}$(E0) is the partial half-life of the initial state for E0 decay and W_{ic} and W_{π} are the transition probabilities for internal-conversion electron and electron-positron pair emission, respectively. They are defined by

$$W_{ic}(E0) + \mathrm{W}_{\pi}(E0) = \rho^2(E0) \text{ x } \left[\Omega_{ic}(\mathrm{E0}) + \Omega_{\pi}(E0)\right]$$

where Ω_{ic} and Ω_{π} are electronic factors independent of nuclear properties. The nuclear structure information is contained in the quantity ρ(E0), defined by

$$\rho(E0) = \frac{\langle f | M(E0) | i \rangle}{eR^2}$$

where R is the nuclear radius and M(E0) is the monopole matrix element, $\Sigma_k e_k r_k^2$. Since there is often an ambiguity in determining the sign of ρ(E0), it is customary to use the square, ρ^2(E0). The reduced E0 transition probability, B(E0), is given by the square of the E0 matrix element,

$$B(E0) = \rho^2(E0)e^2R^4$$

where e is the electric charge.

In Ref [2], in a discussion of E0 transitions between 2+ states, the quantity

$$q_K^2(E0) = \frac{I_K(E0)}{I_K(E2)}$$

where I_k(E0) and I_k(E2) are the K-shell intensities of the E0 and E2 components, respectively, of the 2+ to 2+ transition.

A dimensionless ratio of the E0 and E2 transition probabilities was defined in Ref [3] as

$$X(E0/E2) = \frac{B(E0)}{B(E2)} = \frac{\rho^2(E0)e^2R^4}{B(E2)}$$

As given in Ref [1], X(E0/E2) can be determined experimentally from the relation

$$X(E0/E2) = 2.54 \text{ x } 10^{-6} \text{ x } q_K^2(E0/E2) \text{ x } \frac{\alpha_K(E2)}{\Omega_K(E0)} \text{ x } E_\gamma^5$$

where Eγ is the E2 γ-ray energy in keV.

The experimental monopole strength can be obtained directly if the partial half-life of the E0 transition, $T_{½}(E0)$, is known.

$$\rho^2(E0) = \frac{\ln 2}{T_{1/2}(E0)}[\Omega_K(E0)+\Omega_{L1}(E0)+\ldots+\Omega_\pi(E0)]^{-1}$$

or alternatively from

$$\rho^2(E0) = q_K^2(E0/E2) \text{ x } \frac{\alpha_K(E2)}{\Omega_K(E0)} \text{ x } W_\gamma(E2)$$

if the E2 transition rate, $W_\gamma(E2)$, is known.

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E. Alpha-Decay Hindrance Factors

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1. The Concept of a Hindrance Factor

The probability for α decay depends primarily on two factors:

(a) the difference in the nuclear structure configurations between the parent and the daughter nuclear states
b) the energy of the α particle.

Note: The dependence on energy is very strong. For example, for ground-state to ground-state decays of even-even isotopes, changing the energy from Eα = 4 MeV to Eα = 8 MeV reduces the partial α half-life, by 20 orders of magnitude.

Our main interest in the Nuclear Data Network is in the effects of nuclear structure on α decay (Jπ and configuration assignments); therefore, it is useful to define the concept of α_{HF} which is related to the experimental α intensity Iα, but with the energy dependence (as well as the weaker Z and A dependence) removed. This is quite similar to the procedures adopted for β decay (the use of log ft's) and γ decay (the use of Weisskopf units for reduced transition probabilities).

In even-even nuclei, the strongest α transitions are the g.s. to g.s. transitions (0^+ to 0^+). They range from 65% to over 99% of total α decay. By definition, HF = 1 for these α branches. All other α HF's are calculated relative to the 0^+ to 0^+ HF's. For example, in the mass region A>214, the HF's for 0^+ even-even parents to the 2^+ first excited daughter states vary smoothly as a function of A from 0.9 to 4.0. See Ref [3].

In odd-A and odd-odd nuclei, the HF is defined relative to the HF's for g.s. to g.s. transitions in the neighboring even-even nuclei (see section 3).

2. The Use of α_{HF} in Nuclear Data Evaluations

The α HF's exhibit remarkable regularities [2,3]. These systematic features are the basis for their usefulness in evaluations (again in close analogy to the use of log ft's and reduced transition probabilities as well as of spectroscopic factors). The main uses are for

(a) Jπ and configuration assignments
(b) estimation of unknown α-decay branches

a) Jπ assignments. The Summary of Bases for Spin and Parity Assignments in NDS contains two strong rules, #33 and #34, based on α decay for Jπ assignments; however, more arguments could be suggested based on the systematic trends discussed in Refs [2,3]

It is clear from Ref [2] that in the deformed actinide region, all rotational bands have very characteristic α_{HF}'s. For favored bands i.e., the same configuration in parent and daughter levels (It is assumed that the band are not strongly mixed.), the α_{HF}'s may be calculated easily using the rotational model. The agreement with experiments is usually within a factor of 3 (for L = 2 transitions the agreement is usually better than 50%). Considering the ≈4 orders of magnitude spread in measured α_{HF}'s, this agreement as well as the systematic trends in non-favored transitions (effects of L-transfer, spin flip, Nilsson configuration changes, Coriolis coupling) are very useful for Jπ and configuration assignments. Clearly, our Jπ rules #31 and #32 for rotational bands should be updated since α_{HF}'s are no less useful than level energies in establishing assignment of a level to a rotational band.

The systematic trends in the closed-shell lead region are no less impressive. Ref [3] . For example, α_{HF}'s from parent $3p_{1/2}$ to daughters $3p_{1/2}$, $3p_{3/2}$ and $2f_{5/2}$ are the same to within ≈20% for Po, Rn, and Ra isotopes. Similar agreement is apparent: in the $2f_{5/2}$parents decays to ^{201,203,205}Po; in the$2g_{9/2}$ parents decays to ^{209}Pb, ^{209}Po, ^{211}Rn, ^{213}Ra; and in the $1h_{9/2}$ parents decays to ^{207,209}Tl, ^{207,209,211,213}Bi, and ^{215}At.

The consistency is not as good, but still impressive in the odd-odd nuclei: the decay of $(\pi 1h_{9/2})(\nu 2g_{/2})1^-$ and $(\pi 1h_{9/2})(\nu 2g_{9/2})9^-$ parents. Clearly, our Jπ assignment weak argument #4 can be strengthened when supporting α_{HF} information is available.

b) **Estimation of unknown α decay branches**. The same systematic trends of α decay that were pointed out in Refs [2,3] can also be used to estimate unknown α branches. One type of application is to estimate an intensity of a single α branch which was not measured but may be of importance to the mass chain evaluation. For example, we estimated the ^{209}Po α branch to the $5/2^-$ g.s. of ^{205}Pb at 20%. Experimentally it was not possible yet to resolve this branch from the favored 80% α to the $1/2^-$ state. This estimate is relevant to the calculation of Q_α of ^{209}Po as well as to the degree of usefulness of ^{209}Po as an α energy standard. Another example is the estimate of I_α to 2^+ states in a number of heavy elements based on interpolation of the very smooth variation of α_{HF}'s in this region. This estimate is essential for the correct calculation of the radius parameter used to calculate α_{HF}'s for the whole region (see section 3).

The second type of application is the estimate of %α, i.e., the total α decay branching of ground states or isomers in cases where this branching is not known experimentally. The key to these estimates is the systematics of favored α transitions, which are usually by far the most intense and determine to a large extent the total α-decay branching. (The exceptions are cases where the favored level in the daughter is very high in energy.)

In the case of even-even ground-state α decays, the smooth systematic trends of the radius parameter (section 3) determine the intensity of the main 0^+ to 0^+ α branch. The second strongest transition usually is the 0^+ to first-excited 2^+ state; this branch can be estimated quite reliably from the systematic trend of 2^+ HF's. Often the above 2 branches account for over 99% of the total a decay.

For odd-A and odd-odd nuclei, the estimates of %α can be quite reliable provided that the level energy of the favored configuration in the daughter is known. In odd-A nuclei, such estimates may be reliable to ±20% when Qα is well known. In odd-odd nuclei (where less good data are available), the reliability may be ±50%. For example, in the mass region A = 191 through 213 for all 50 cases of odd-A favored α's, HF's vary from 1.1 to 1.6 for $J \neq 1/2$ and from 1.4 to 2.2 for J=1/2. For odd-odd nuclides the favored α_{HF}'s vary from 1.5 to 2.5 (except for 5^+ states which are probably of mixed configuration). In transition regions (where the deformation changes rapidly), there are significant differences between the parent and the "favored" daughter configurations; as a result, the "favored" α_{HF}'s are larger.

Uncertainties in Qα of 200-400 key correspond to an order of magnitude uncertainty in a calculated $T_{½}(\alpha)$. Even in cases of such large uncertainties, the estimate of %α may still be useful. For example, the estimate %α << 1 syst may indicate that %ε ≈ 100; thus, log ft's could be calculated. Table 5 of Ref [3] lists the %α and Qα values for $186 < A < 223$; "syst" indicates the values derived from systematics of α_{HF} and of Qα, respectively. For a graphical representation of Qα values see, for example, Ref [4]. In a few cases, $T_{½}$ could be estimated for ground states and isomers. Table 6 of Ref [3] lists the individual α_{HF} values, including the ones derived from the systematic trends, and the Iα values deduced from the α_{HF} values. In cases of strong configuration mixing, the estimates are less reliable; however, for strong α branches, the sensitivity of α_{HF} (and therefore of Iα) to configuration mixing is much smaller than the corresponding sensitivity of log ft values, of reduced transition probabilities, and in many cases of magnetic moments.

3. Calculation of α_{HF}*

The calculation of $\alpha_{HF's}$ in NDS is based on the spin-independent equations of Preston, Ref [5], and is essentially the same as the calculations done for the sixth, seventh and eighth editions of the Table of Isotopes, Ref [6] (1967, 1978, and 1996).

a) For even-even nuclei, the α_{HF}'s of excited states are inversely proportional to Iα and are normalized to the value HF = 1 for the 0^+ to 0^+ g.s. to g.s. transition. The computer program removes the energy dependence (which is calculated from the input Qα and E_{level} in the daughter). The uncertainties in the parent $T_{½}$, α branching, and Qα cancel out, because of the method of normalization. If the level energies are accurate (say, $\Delta E \leq 5$ keV), the uncertainty in HF will be the same as the fractional uncertainty in Iα.

* See **Appendix A** for details on obtaining the r_0 parameters

In addition to α_{HF}, the computer program calculates the parameter, r_o, (roughly equivalent to the nuclear radius) from Q_α, $T_{½}(\alpha)$, and I_α to the g.s. (Z and A also enter in). It is useful for evaluators to keep track of the r_0 systematics in the region of their responsibility. In my experience (in the lead and actinide regions) the ro values for each element lie on fairly smooth curves, the exception being sharp breaks at the closed shells N = 126 and N = 152.

b) For odd-A and odd-odd nuclei, the α_{HF}'s are also inversely proportional to I_α, but the normalization is to the neighboring nuclei. The r_o parameter has to be included in the input to the computer program. Usually for odd-A isotopes, the r_o will be the average of the two nearest even-even neighbors, and for odd-odd isotopes the average of the four nearest even-even neighbors. See **Appendix A**. If the r_0's for some (or all) of the neighbors are not known, then interpolation or extrapolation is needed. Our experience at Oak Ridge is that human interpolations (or extrapolations) are preferable to computer algorithms for this purpose. The uncertainties in α_{HF} are usually much larger than in the case of even-even nuclei for the following reasons: There is the additional uncertainty in the r_0 parameter; the uncertainties in Q_α and $T_{½}$ (parent) as well as in the α-branching of the parent, do not cancel out. Typical uncertainties are, for example, 3 keV in Qα of 5 MeV resulting in ≈4% uncertainty in α_{HF}, Δr_0 of 0.01 resulting in ≈20% uncertainty in α_{HF}, and of course linear dependence on uncertainties in I_α and $T_{½}(\alpha)$.

In contrast to most calculations of log ft's, we do not have to worry here about unplaced transitions. In fact, α_{HF}'s can be calculated from E_α, I_α and Z without any knowledge of the decay scheme. However, the interpretation of the results of α_{HF} calculations demands considerable experience and detailed knowledge of nuclear structure.

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F. Jπ and Multipolarity Assignments in (HI,xnypzαγ) Reactions

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In heavy-ion compound nuclear experiments (HI,xnypzαγ), the multipolarities of the γ transitions and the relative spins and parities of the levels are generally determined through the measurement of angular distributions, angular correlations, linear polarizations, and through measurements of internal conversion coefficients.

Angular distributions

1. The angular distributions of γ-rays, W(θ), is a measurement of the intensity as a function of the angle θ with respect to the beam direction or to the nuclear spin axis.

$$W(\theta)=\sum_{k=\text{even}} A_k P_k(\cos\theta)$$

2. The values of the coefficients A_k depend on ΔJ, the mixing ratio δ(L+1)/L, and the degree of alignment. In most cases only A_2 and A_4 need to be considered. For high-spin states the distributions are largely independent of spin.

3 The degree of alignment, σ/J, is usually determined through a measurement of W(θ) for a number of known ΔJ =2 transitions. In actual practice many authors use σ/J = 0.3 for the degree of alignment. Here σ is the half-width of the Gaussian describing the magnetic sub-state population. The attenuation caused by the degree of alignment affects only the magnitudes of A_2 and A_4. Level lifetimes are assumed to be small so that alignment is maintained.

4. Angular distribution measurements alone may be used to deduce ΔJ and/or the multipole order (L) (multipolarity), but not Δπ.

5. Typical values of A_2 and A_4 are given in the table below (σ/J = 0.3 assumed). The angle θ is measured relative to the beam direction. If θ were with respect to the spin axis, then the sign of A_2 is generally reversed.

Table of Angular Distributions

ΔJ	Multipole Order	Sign of A_k*		Typical Values	
		A_2	A_4	A_2	A_4
2	Quadrupole	+	-	+0.3	-0.1
1	Dipole	-		-0.2	0.0
1	Quadrupole	-	+	-0.1	+0.2
1	Dipole + Quadrupole	+ or -	+	+0.5 to -0.8	0.0 to +0.2
0	Dipole	+		+0.35	0.0
0	Quadrupole	-	-	-0.25	-0.25
0	Dipole + Quadrupole	+ or -	-	+0.35 to -0.25	0.0 to -0.25

* Note that A_4 is positive for all δ values for $\Delta J=1$, Dipole +Quadrupole, and negative for $\Delta J=0$, Dipole + Quadrupole.

Angular Correlations, e.g. DCO, Directional Correlations of γ-rays from Oriented States of Nuclei

6. DCO measurements involve the determination of the coincidence intensities for two γ-rays, γ_K of known multipole order (L) and γ_U of unknown multipole order. The γ-rays are detected at two angles, θ_1 and θ_2, with respect to the beam direction. The coincidence intensities are determined as two dimensional areas, $I(\theta_1\ \theta_2\ \gamma_K\ \gamma_U)$ and $I(\theta_1\ \theta_2 \gamma_U\ \gamma_K)$, where in the former case γ_K is detected at angle θ_1 and γ_U at angle θ_2.

The DCO ratios are then defined as

$$R=I(\theta_1\theta_2\gamma_k\gamma_u)/I(\theta_1\theta_2\gamma_u\gamma_k)$$

7. As with angular distributions, these ratios are insensitive to spin for high spin states but are sensitive to relative spins and γ multipole orders.

8. The angles θ_1 and θ_2 are generally determined by the geometry of the array. The values of R given below are typical for an array with detectors at 37° and 79°. An alignment of $\sigma/J=0.3$ has been assumed.

Table of Typical DCO Ratios

ΔJ_γ^{gate}, Multipole order	ΔJ_γ	Multipole order	Typical R(DCO)
2, Quadrupole	2	Quadrupole	1.0
2, Quadrupole	1	Dipole	0.56 (θ_1=37°, θ_2=79°)
2, Quadrupole	1	Dipole + Quadrupole	0.2 to 1.3 (θ_1=37°, θ_2=79°)
2, Quadrupole	0	Dipole	1.0
2, Quadrupole	0	Dipole + Quadrupole	0.6 to 1.0 (θ_1=37°, θ_2=79°)
1, Dipole	2	Quadrupole	1/0.56 (θ_1=37°, θ_2=79°)
1, Dipole	1	Dipole	1.0
1, Dipole	0	Dipole	~1/0.56

Linear polarization of γ-rays

9. A Compton polarimeter apparatus allows the measurement of relative intensities of radiation scattered in planes perpendicular to and parallel to the reaction plane (plane defined by the beam direction and incident gamma ray).

10. Determination of γ-ray polarization may differentiate between electric and magnetic radiations and thus, when combined with correlation data, allow determination of Δπ. See Kim et al. [7]

Internal conversion coefficient data

11. Internal conversion coefficients or subshell ratios may be obtained from electron spectra or from γ-ray intensity balances.

12. The interpretation of internal conversion coefficient data is as given in NDS under rules for spin and parity assignments. Note that electron data usually give K-, L-, etc., conversion coefficients or sub-shell ratios whereas intensity balance arguments give total conversion coefficients

Other considerations

13. If T½ (level) is known or a limit can be assumed (based on coincidence resolving time, for example), RUL (recommended upper limits for Weisskopf estimates) may serve to eliminate the M2 option for a $\Delta J = 2$ quadrupole transition.

14. Generally for the states populated in high-spin reactions, spins increase with increasing excitation energy. This is a result of the fact that these reactions tend to populate yrast or near yrast states.

15. For a well-deformed nucleus, when a regular sequence of $\Delta J=2$ (stretched quadrupole) transitions is observed at high spins as a cascade, then the sequence may be assigned to a common band with E2 multipolarity for all the transitions in the cascade. A similar but somewhat weaker argument holds for less deformed nuclei where a common structure of levels is connected by a regular sequence of $\Delta J=2$ (stretched transitions) as a cascade. For interband transitions, $\Delta J = 1$ or 0 transitions with significant admixtures are considered to be of MI + E2 type. If the transition is pure dipole ($\delta(Q/D)=0$), it is quite often El. The small deformation magnetic rotational (M1) bands present exceptions to this rule.

16. The presence of strongly coupled (deformation alignment) bands allows assignment of relative spins and parities of the band members. The presence of a measurable quadrupole admixture in the $\Delta J = 1$ cascading transitions is required to prove that all the states have the same parity. This is because nuclei with octupole deformation may have two rotational $\Delta J = 2$ sequences of opposite parity connected by cascading El transitions.

17. For near-spherical nuclei, when a regular sequence of $\Delta J = 1$ (stretched dipole) transitions is observed at high spins as a cascade, then the sequence may be assigned to a common band with M1 multipolarity for all the transitions in the cascade. (Cascades of $\Delta J = 1$, El transitions occur in rare cases of nuclides which show alternating-parity bands or reflection asymmetry.)

18. In the absence of angular distribution/correlation data, a regular sequence of transitions in a cascade may be assigned to a common structure or a band if:

1) The low-lying levels of this structure have well established spin parity assignments.

2) There is good evidence that, at higher energies and spins, the band has not changed in its internal structure due to band crossings or other perturbations.

19. In strongly coupled bands, (deformation aligned) a comparison of an experimentally deduced value of g_K (from mixing ratio $\delta(E2/M1)$ and assumed g_R and Q_0) with that calculated on the basis of a proposed quasi-particle configuration may lead to the assignment of parity to a band.

20. A comparison of experimental and calculated Routhians and particle alignments (from cranked shell-model calculations) for suggested quasi-particle configurations may give information about the parity of a rotational band.

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G. Single-Nucleon Transfer Reactions

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The following remarks on single-nucleon transfer reactions are hopefully useful to A-chain evaluators. For some more details see the introduction of the paper: P. M. Endt "Spectroscopic factors for single-nucleon transfer in the A=21-44 region", *Atomic Data and Nucl. Data Tables* 19 (1977) 23.

For the time being it does not seem advisable to list S-factors from two or more-nucleon transfer reactions in the NDS. For such reactions it is not possible to factorize the cross section into a nuclear structure part and a part relating to the reaction mechanism. Spectroscopic information from single-nucleon transfer reactions in which the in- and outgoing particles are heavier than the α-particle should also be excluded, because as yet the reaction mechanism for such reactions is far from being established.

Finally, S-factors from work performed at either too low or too high a bombarding energy, or with poor resolution should not be listed. At low bombarding energy ($E \lesssim 5$ MeV) the compound-nucleus contribution is relatively large. Especially for rather light nuclei the Hauser-Feshbach theory is not considered good enough to predict the magnitude or the angular distribution of this contribution with confidence. At high bombarding energy ($E \gtrsim 50$ MeV) the incoming particle penetrates deep into the nucleus, which entails changes in the optical potentials on which, at present, too little systematic information is available. Poor resolution (FWHM $\gtrsim 100$ - 200 keV) not only reduces the number of resolved particle groups but, perhaps worse, makes it more difficult to recognize contaminant groups and to subtract their contribution. Contaminant groups are characterized by their energy changes as a function of angle and/or bombarding energy; for adequately accurate energy measurements, good resolution evidently is a necessity. In this respect, work performed with magnetic spectrometers is generally superior to that with semi-conductor detector telescopes.

The following notation has proved practical for the spectroscopic factors relevant to the four different types of single-nucleon transfer reactions:

S_n^+ neutron stripping (d,p),(t,d),(α, τ);

S_p^+ proton stripping (d,n),(τ, d),(α, t);

S_n^- neutron pick-up (p,d),(d,t),(τ, α);

S_p^- proton pick-up (d, τ),(t, α).

Poor resolution generally excludes work with the (n,d) reaction. The distorted-wave Born approximation (DWBA) theory for the analysis of differential cross sections for direct single-nucleon transfer reactions has certainly been very successful. A vast number of l-determinations have greatly furthered our knowledge of Jπ values. It is also true, however, that the theory is not as perfect as, say, that for γ- γ angular correlations. Contributions from multistep processes (calculated with the coupled-channel formalism) and from compound-nucleus formation exist and are often evaluated quantitatively, but the reliability of such calculated corrections is not yet fully known. Uncertainty also exists in the values of the optical-model parameters to be used, in the parameters determining the bound-state radial wavefunctions of the transferred particle, and in finite-range and nonlocality corrections. One may apply a least-squares analysis to measured angular distributions, but χ^2-values close to unity are still, at least for good statistics, a dream of the future. The correctness of l-values is still judged by eye, and consequently only very few l-assignments are unambiguous, in the sense that other l-possibilities can be excluded at the 0.1% probability limit.

The difficulties mentioned above are even more important for the spectroscopic factors extracted from a DWBA analysis. Few authors assign errors to spectroscopic factors because in most cases these would be of a systematic rather than of a statistical nature. It is thus difficult to compare the results of two measurements (the definition of good or bad agreement depends on the errors) or to compare measured and theoretical values.

The measured differential cross section $\sigma(\theta)_{\exp}$ and the theoretical differential cross section $\sigma(\theta)_{\mathrm{DWBA}}$ provided by a DWBA program are related as follows:

$$\sigma(\theta)_{\exp} = NC^2 S^- \sigma(\theta)_{DWBA} \text{ for pickup,} \tag{1}$$

and

$$\sigma(\theta)_{\exp} = NC^2 \frac{(2J_f+1)}{(2J_i+1)} S^+ \sigma(\theta)_{DWBA} \text{ for stripping} \tag{2}$$

This is true, for example, for JULIE, but we note that the output of DWUCK, the most widely used program, is slightly different:

$$\sigma(\theta)_{DWBA}^{DWUCK} = (2j+1)\sigma(\theta)_{DWBA}^{JULIE},$$

where j is the total angular momentum of the transferred nucleon. In these expressions C^2 denotes the (squared) isospin Clebsch-Gordan coefficient for single-nucleon transfer

$$C = \langle T_i T_{zi} 1/2 \pm 1/2 \,|\, T_f T_{zf} \rangle,$$

where (Ti,Tzi) relates to the initial (target) nucleus and (Tf,Tzf) to the final state. The C^2 values can be evaluated with the help of Table 1. It shows, for example, that one has $C^2 = 1$ for neutron stripping. It should be remarked that in many papers published before about 1970 the S-values have to be interpreted as C^2S.

The normalizing factor N is proportional to the square of the overlap integral between (for stripping) the wavefunctions of the outgoing particle coupled to the transferred nucleon and that of the incoming particle. For pick-up the words "ingoing" and "outgoing" in the preceding sentence have to be interchanged.

One can consider N as the spectroscopic factor for the light particles participating in the reaction. Whereas, in a transfer reaction A(a,b)B, the spectroscopic factor measures the wavefunction overlap

between A and B and the transferred nucleon, the factor N has the same function for a and b and the transferred nucleon. Numerical values of N for some reactions are given in Table 2.
Spectroscopic factors can be subjected to several tests. First one can check that reactions of one type, such as (d,n) and (^{3}He,d) (proton stripping), or (p,d), (d,t) and (^{3}He, α) (neutron pick-up), produce the same spectroscopic factors. The same should hold for pairs of reactions, such as (d,p) and (^{3}He,d), or (p,d) and (d,^{3}He), exciting mirror states, or, more generally, components of the same isospin multiplet. Finally, one can check the equality of spectroscopic factors for pairs of inverse reactions, for example, (d ,p) and (p ,d), connecting ground states of stable nuclei. Because the ratios of spectroscopic factors found for the pairs of reactions mentioned above are reasonably close to unity , one may conclude that the set of normalization constants used is internally consistent.

The experimentally observed deviations from these consistency rules provide some ideas as to the experimental errors in S-factors. For absolute measurements the error may be taken as 25% . Relative measurements are presumably more accurate, in particular for groups of S-factors relating to the same l-value.

The credibility of published l-values depends not only on statistics and on the number of points in the angular distributions, but also on the l-value itself . In the sd shell, values l>3 have proven to be quite unreliable, and the same presumably holds for l >4 in the fp shell. Generally, high l-values (like l=4) should be mistrusted if the author does not show explicitly that the DWBA curves for l=3 and l =5 are sufficiently different from that for l=4 . A reaction like (τ,α) yields relatively unstructured angular distributions and thus leads to unreliable l-values.

Spectroscopic factors cannot be arbitrarily large because they are subject to sum rules. The sum rules useful for the derivation of upper limits are the following:

$$\sum \frac{2T_f}{2T_f+1} S_p^- =< p > \tag{3}$$

and

$$\sum \frac{2J_f+1}{2J_i+1} S_n^+ =< n^{-1} > \tag{4}$$

where $< p >$ is the number of protons and $< n^{-1} >$ the number of neutron holes in a subshell (n, l, j), both in the target nucleus. The summation has to be extended over all final states (whatever the spin) which can be reached by transfer of a particle in the subshell (n, l, j). For the proton pick-up and neutron stripping considered here one can only reach final states with isospin $T_f = T_i + ½$. Equation (3) also holds for neutron pick-up and Eq. (4) for neutron stripping (both right-hand members unchanged) if the summation is extended over $T_f = T_i + ½$ states only; in these cases the reaction can proceed to both $T_f = T_i + ½$ and $T_f = T_i - ½$ final states (if at least Ti > 0).

We shall use here Eqs. (3) and (4) only for even-even target nuclei, corresponding to Ji = 0, J_f = j. From the fact that neither $< p >$ nor $< n^{-1} >$ can exceed 2j + 1, one then obtains for this case the upper limits for any single transition

$$S_p^- \leq \frac{2T_f + 1}{T_f}(2j+1) \tag{5}$$

and

$$S_n^+ \leq 1. \tag{6}$$

The more complicated rules for proton stripping and for neutron pick-up reactions leading to $T_f = T_i - ½$ states are not mentioned here because their applicability is very limited.

Table 1
Isospin Clebsch-Gordan coefficients (C) for use in single-nucleon transfer reactions†

	Stripping $(2J_i + 1)C^2$		Pick-up $(2J_f + 1)C^2$	
	p	n	p	n
$T_f = T_i - ½$	$2T_i$	0	0	$2T_f + 1$
$T_f = T_i + ½$	1	$2T_i + 1$	$2T_f$	1

† T_i and T_f denote the isospins of the target nucleus and the final state, respectively.

Table 2
Normalizing factors (N) for single-nucleon transfer reactions

Reaction	N	Reaction	N
(d,p), (d,n)	1.53	(p,d)	2.29
(τ,d)	4.42	(d,τ)	2.95
(α,τ)	46	(τ,α)	23
		(d,t)	3.33

These N-values fulfill the relation for inverse reactions $N(b,a) = N(a,b)\frac{2s_a + 1}{2s_b + 1}$, where s_a and s_b are the spins of the particles concerned.

H. Nuclear Structure and Decay Data Evaluation Procedures and Guidelines for Strongly Deformed Nuclei

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I Introduction

To assist the evaluator of Nuclear Structure and Decay Data for the Nuclear Data Sheets, as well as to help provide some degree of uniformity and consistency in the resulting evaluations, numerous guidelines have been established over the years. Many of these are categorized as being either "strong" arguments or "weak" arguments for making Jπ assignments to energy levels in nuclei. For those levels that are members of rotational bands in strongly deformed nuclei, however, the establishment of "strong" or "weak" rules for making Jπ assignments is at one and the same time a trivial task and a complex one. On the one hand, implicit in even considering that a level might belong to a rotational band is that a model-based judgement is being made, taking into account other information in addition to that which is explicitly being evaluated. Such "external" information includes, for example, the observation that a relatively simple relationship exists between the energy and Jπ value of the particular level and those of certain other levels in the same nucleus and that similar patterns, presumably analogous to the case at hand, are observed in other, near-lying, nuclides.

On the other hand, as regards Jπ values, members of rotational bands are not really different from levels in nuclides that are not strongly deformed, in that the usual means of assigning such values (γ-decay and feeding properties, α and β-feeding probabilities, γ-γ directional correlations, etc.) still apply. It was, in fact, only after definitive Jπ values had been assigned to certain groups of levels (and strongly enhanced connecting E2 transitions observed) that the occurrence of the hypothesized rotational bands could be established in the first place.

It should be recognized that reliable Jπ assignments to levels in the strongly deformed nuclei can frequently be made primarily by using nuclear model-based considerations. For example, the existence of rotational bands in many nuclides is extremely well established; and the models of the intrinsic (i.e., nonrotational) states upon which they are built are relatively simple and, also, quite well understood (at least as regards Jπ values). In addition, the angular-momentum coupling scheme underlying this picture is simple and can usually be applied to actual nuclear level schemes without the use of complex, computer-based nuclear structure calculations. These considerations, together with the existence of an extensive "systematics" of level properties of the strongly deformed nuclei make it possible in many cases for the experienced nuclear physicist to construct a level scheme for a previously unstudied strongly deformed nuclide in which the Jπ assignments can be regarded as reliable, even though the available data are sufficiently meager that, in other mass regions, they would yield almost no insight into the Jπ values.

It should further be noted that these judgments are frequently based on **qualitative** considerations rather than **quantitative** ones (such as, for example, the magnitude of a log ft value or the lifetime of a γ-ray transition). Consequently, these inferred Jπ values cannot be regarded as being based on "strong" arguments, as this term is employed in Nuclear Data Sheets evaluations, even though they are reliable as far as the underlying nuclear theory is concerned. Thus, for the strongly deformed nuclei, the traditional distinction between "strong" and "weak" arguments for Jπ assignments becomes blurred. In these cases, then, the assignment of Jπ values ultimately comes down, as it always should, to the judgment of the individual evaluator.

In the discussion below, we lay out some considerations to help guide the evaluator in his or her evaluation of nuclear-structure data to provide Jπ assignments for levels in the strongly deformed nuclides. The topics treated involve only those aspects of the data that are specific (or unique) to the angular-momentum coupling schemes appropriate to these nuclides. Our thinking here is guided by those features of the strongly deformed nuclei that are commonly encountered in the "rare-earth" region (say, 150<A<190) and the actinide region (say, A>220). However, the concepts should be broadly relevant to those other regions of the Nuclide Chart where, as indicated by an increasing body of experimental data, strongly deformed nuclei also occur. It is assumed that the reader is familiar with those considerations for making Jπ assignments (such as γ-ray multipolarities) that are independent of the features of any specific nuclear coupling scheme; and, although implicit use is made of these, no explicit elaboration of them is given.

In Section II, we give several considerations to be kept in mind in treating data on the strongly deformed nuclides. In Section III, to further elucidate some of these ideas, we provide a summary of the analysis of a specific case, namely the strongly deformed, presumably reflection-asymmetric, nuclide ^{225}Ra [1]. Finally, in an Appendix we give a concise summary of the ideas presented in these three Sections.

For further reading on nuclear-model considerations as applied to the analysis of the level structure of strongly deformed nuclides, the following references are recommended. Quite instructive, although somewhat old, reviews are those of Mottelson and Nilsson [2] and Gallagher and Soloviev [3]. Comprehensive evaluations of the then-available data on the odd-mass nuclides in the rare-earth and the actinide regions, respectively, are given in Refs. [4] and [5]. The level schemes of the even-A nuclides in the rare-earth region are interpreted, and the underlying nuclear theory presented, in Ref. [6].

II Selected Properties of Rotational Bands

A. Level Energies

1. Low Rotational Frequencies and Weak Band Mixing

The rotational spectra of strongly deformed nuclei at low frequencies of rotation are customarily analyzed using the well known expression (see, e.g., Refs. [4, 7, 8,])

$$E(J,K) = E_K + AX + BX^2 + CX^3 + \text{etc.} + (-1)^{J+K} \prod_{i=1-K}^{K} (J+i)(A_{2K} + B_{2K}X + \text{etc.}) \qquad (1)$$

where *X* represents either J(J+1) or J(J+1)-K^2.

In the present discussion, we choose the latter expression for X. K denotes the projection of the total angular momentum of the intrinsic state on which the band is built onto the nuclear symmetry axis.

In applying eq. (1) to the analysis of level energies within a given rotational band, one typically works with level-energy differences only, and thus the parameter E_K (which serves to locate the energy of the band head) can be neglected. For bands with K = 0, the terms in eq.(1) with alternating signs vanish, while, for bands with K = 1/2 and 1, one has explicitly

$$\begin{aligned} E_J &= E_K + AX + BX^2 + \text{etc.} && \\ &+(-1)^{J+\frac{1}{2}}(J+\tfrac{1}{2})(A_1 + B_1X + \text{etc.}) && \text{for } K=\tfrac{1}{2} \qquad (2,a) \\ &+(-1)^{J+1}J(J+1)(A_2 + B_2X + \text{etc.}) && \text{for } K=1 \qquad (2,b) \end{aligned}$$

Corresponding expressions can be derived from eq. (1) for bands having larger values of K. With the identification $A_1 = A\underline{a}$, where a denotes the decoupling parameter, eq. (2,a) can readily be expressed in the usual form for K = 1/2 bands, viz.

$$E(J,\tfrac{1}{2})=E_{\frac{1}{2}} + A[J(J+1) - \tfrac{1}{4} + (-1)^{J+\frac{1}{2}}(J+\tfrac{1}{2})\,\underline{a}] \qquad (3)$$

In the derivation of the relationship expressed in eq. (1), it is assumed that K is, at least approximately, a good quantum number. This implies that the coupling (mixing) of the band under consideration to (with) other bands in the same nucleus is not too strong and that the rotational frequencies of the states are not too high. In such cases, the coefficients B and C are expected to be small (e.g., $B/A \sim 10^{-3}$ and $C/B \sim 10^{-3}$) and a reasonably good description of the energies of the band can usually be provided using only a few parameters (e.g., A, B and, for K=½ bands, a).

Although explicit expressions can be derived [8, 9, 10] relating several of the parameters in eq.(1) to the matrix elements assumed to couple the band in question to the other bands in the nucleus, such computations usually lie beyond the scope of the typical A-chain evaluation. Rather, the usefulness of giving values for these parameters in an A-chain evaluation lies in providing the interested reader with a rapid and convenient means of gaining information about the band. For example, for K =½ bands, the decoupling parameter provides almost unique information about the nature and extent of the single-particle (or one-quasiparticle) content of the band. Similarly, the rotational constant A (= $\hbar^2/2\mathfrak{I}$) gives information about the effective moment of inertia ($\mathfrak{I}$) of the band; and an extensive systematics of the A-values for bands in the strongly deformed nuclei exists (see, e.g.,Refs.[4, 5]).

The other parameters also play an important role in helping the reader "understand" the band. The values of the parameters A_{2K} (and B_{2K}) give a measure of the "staggering"† (signature splitting) within the band and hence can be informative. The magnitudes of these parameters are expected to decrease rapidly with increasing K-value and, hence, their effects should be most readily apparent in those bands having the smaller values of K. Unfortunately, in most of the evaluations of the level schemes of the strongly deformed nuclei such terms have not been considered. We would encourage evaluators to include them, where appropriate, in their future work.

The analysis of the level energies of a given rotational band to deduce realistic values for the band

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† For even-A nuclides, a relative displacement of the odd-spin band members with respect to those of even spin. For odd-A nuclides, a relative displacement of the band members for which J+½ = even with respect to those for which J+½ = odd.

parameters is not always a trivial task or an obvious procedure. In doing this, the following points should be kept in mind.

(i) Eq.(1) is useful in describing rotational bands only when the number of parameters needed to describe the level spacings is small. Since it is in reality an expansion in powers of $J(J+1)-K^2$, rather than a closed expression, it is possible to fit "exactly" the energies of an arbitrarily large number of band members simply by including a correspondingly large number of terms. However, such a procedure would not produce physically meaningful results beyond those obtained from fitting a few terms, and would most likely have rather little predictive power (i.e., ability to predict the energies of the next levels).

(ii) Consequently, one should generally try to use the smallest number of parameters consistent with achieving a reasonable overall fit to the level energies. These parameter values should be determined from the smallest possible number of the lowest-spin members of the band (recognizing that the energies predicted for the higher-spin band members may then differ somewhat from the observed values). In particular, in most cases it is probably not meaningful to carry out a least-squares analysis of the energies of a rotational band in order to obtain a set of "best" values for the band parameters.

(iii) Careful attention needs to be given to the choice of which parameters are chosen to give the "best" description of the band. The parameter A, of course, (and, for K=1/2 bands, a) should always be included but, beyond this, the choice is not always clear. If only a small number of band members are known, and the customary choice of A and B to describe the band leads to unreasonable results (e.g., the contribution of the "B term" to the level energies is comparable to that of the "A term"), the deduced parameters are not meaningful and thus should not be quoted. In these situations, one should repeat the fit using A_{2K} instead of B and compare the quality of the results from the two fits. If this latter fit appears to provide a "reasonable" description of the level energies, those deduced parameters can be given; otherwise it is perhaps better to list no parameter values (and to point out this fact).

(iv) In some cases, the differences among the sets of parameter values derived from the use of different combinations of level energies are large. These can occur, for example, where the coupling to other bands is strong (and the assumptions on which eq.(1) is based are thus not valid) or where the band parameters being used to describe the band are not the best ones. In these latter cases, it is again important to consider choosing different parameters keeping A (and, where K = ½ bands are involved, a) in an attempt to get a better description of the band.

2. Low Rotational Frequencies and Strong Band Mixing

There are a number of situations in which application of eq.(1) to determine rotational parameters for a band yields "unreasonable" values. These include those in which the bands are strongly Coriolis mixed with other bands. These strongly coupled bands are associated with the low K-value orbitals originating from the "unique-parity" spherical shell-model states, namely the $i_{13/2}$ neutron state and the $h_{11/2}$ proton state in the rare-earth region and the $j_{15/2}$ neutron state in the actinide region. Also included among such bands are some of the K=0 and 1 octupole vibrations in the rare-earth region (the two-quasiparticle makeup of these excitations contains significant contributions from these

unique-parity orbitals).
In these cases, the use of eq.(1) to describe the rotational properties of the band is not justified. To treat them correctly requires the carrying out of a detailed analysis of the Coriolis mixing. While such analyses have proven quite successful in describing even rather unrecognizably distorted rotational band structures (see, for example, [11-13]), they are usually quite time-consuming and lie outside the customary scope of a mass-chain evaluation. Here, though, the evaluator can use the existence of the strong distortion of the band structure as evidence for the presence of strong Coriolis mixing and hence of the intrinsic configurations involved. This knowledge alone can frequently serve as a guide in the choosing of reliable Jπ assignments for the levels.

3. High Rotational Frequencies

The focus of the discussion thus far has been on situations where K is, at least approximately, a good quantum number and eq.(1) applies, i.e., the rotational band structure at low energies, the energy region historically explored by radioactive-decay studies and nuclear reactions initiated by relatively low-energy projectiles. Here the basic nuclear model involves individual particle (or, quasiparticle) or collective motion in a slowly rotating, deformed nuclear potential well.

With the availability of high-energy beams of heavy ions it has become possible to produce and study nuclear systems possessing very large amounts of angular momentum. This has led, over the past decade or so, to the production of an extensive body of information on the properties of rotational bands up to quite high spins. An excellent review of this subject is given in Ref.[14].

In many cases it has been possible to connect the high-spin band structures with their lower-spin portions, previously established using the more conventional techniques. In some instances, one observes already at relatively low spins sizeable departures from a simple J(J+1) spectrum and the splitting up of the band into two distinct bands, one having J+½ = odd and the other having J+½ = even. However, in other instances, a much more normal band structure (i.e., approximately J(J+1) with a relatively small amount of such "staggering") is found to persist up to rather large spin values.

There is a tendency for the evaluator to apply to these higher-spin states the same nuclear-model considerations that are customarily applied to those band members that are located near the band head. However, such an approach is neither correct nor meaningful and, if applied strictly, can lead to unphysical conclusions. Among the reasons for this is the fact that, as the rotational frequency increases, K ceases to be a good quantum number. The Coriolis effects, which can be either neglected or incorporated as "small" corrections to the rotational motion at low frequencies, now become dominant. These significantly affect the band structure in a number of ways, for example, through the occurrence of backbending. Further, the identification of the higher-spin states with a specific Nilsson orbital is not especially meaningful, since these states are in general expected to contain comparable contributions from a number of such orbitals.

For a description of these states, the appropriate symmetry operation is rotation of the nuclear system through 180^0 about an axis (x-axis) perpendicular to the nuclear symmetry axis (z-axis) [14]. The associated quantum number is denoted as the "signature", r, which, together with the parity, provides a means of classifying the nucleonic states in a rotating nuclear potential. More commonly used for this purpose, instead of r, is a quantity $\alpha^\dagger$, where α is defined through the relation $r = e^{-i\pi\alpha}$.

The following relations exist between α and the total angular momentum, J:

$\alpha=0$ $(r = +1)$, $J=0, 2, 4$, etc.
$\alpha=1$ $(r = -1)$, $J=1, 3, 5$, etc.
$\alpha=+\frac{1}{2}$ $(r = -i)$, $J=1/2, 5/2, 9/2$, etc.
$\alpha=-\frac{1}{2}$ $(r = +i)$, $J=3/2, 7/2, 11/2$, etc.

How, then, should the evaluator proceed in dealing with these situations? As regards the experimental situation, since essentially all the data currently available on these states come from in-beam γ-ray (and, occasionally, conversion-electron) spectroscopy, there are several things that remain unchanged. First, the arrangement of the observed energy levels into rotational bands can still be carried out with considerable confidence, based on their γ-decay patterns, when these γ-ray placements are supported by coincidence data. Second, where γ-ray angular distribution data exist and cover a sufficient number of angles that the distribution function can be considered to be well determined, there exists a reasonable basis for assigning $J\pi$ values. The angular distribution functions for "stretched" quadrupole transitions (i.e., $\Delta J = 2$) and "stretched" dipole transitions ($\Delta J = 1$) are distinctive and when these are observed the appropriate spin **differences** (2 and 1, respectively) can be regarded as being well established. [However, the angular distribution for a dipole transition with no spin change ($\Delta J = 0$) has the same form as that of a stretched quadrupole and one must thus be careful to consider this possibility.] Where the experimental situation is such that the high-spin band structure cannot be reliably tied in with its lower-spin counterpart (where, for example, the connecting transitions are low in energy and unobserved or where there is uncertainty in the placement of these γ rays), then only the **relative** energies and $J\pi$ values can be established. Their absolute values are not established, and the whole high-spin band structure must be left "floating" in the level scheme. On the other hand, where these connections to the (presumably) well established lower-spin band members are firmly established, then the energies and $J\pi$ values for all the band members can be determined.

From the theoretical point of view, however, the fact that the nuclear-structure considerations are different at high spins than they are at the lower spins means that the evaluator must cope with a certain degree of ambiguity. While it is possible, and useful, to quote in the customary fashion values for the band parameters and to make nucleonic-configuration assignments to describe the band head and the low-frequency portion of the band, these are generally inappropriate for discussing the higher-spin states. Furthermore, the transition from the regime of spins (or, perhaps better, rotational frequencies) where one coupling scheme is useful to that where the other is more appropriate is not a sharp one. It seems best, therefore, to adopt the following approach in cases where enough of the band structure is established that both low-spin (at and/or near the band head) and high-spin members of a rotational band are observed. The nucleonic configuration (e.g., Nilsson orbital, two-quasiparticle configuration, vibrational excitation) that is believed by the evaluator to best describe the band head should be given, together with the appropriate set of rotational-band parameters. These latter should be those believed to be the most appropriate for description of the energy relationships among the low-lying members of the band and should, of course, be derived from the energies of a small number of these states. Those states used to determine these parameter values should be indicated. In addition, the values of the signature parameter, α, and the parity appropriate for each band member should be given. This could conveniently be done by providing two separate band-(or configuration-) assignment footnotes for each such band. These would list not only the intrinsic configuration assigned to the band but also which of the two possible signature values was appropriate for the various states.

B. Strongly Deformed Nuclides with Reflection-Asymmetric Shapes

In the discussion thus far, it has been assumed that the strongly deformed nuclei under consideration possess equilibrium shapes that are symmetric under reflection in a plane (xy) perpendicular to the nuclear symmetry axis (z). This shape is thus described by deformations of even multipole order (quadrupole, hexadecapole, etc); and it is well established that this assumption is correct for the vast majority of the strongly deformed nuclides. Recently, however, evidence has confirmed the theoretical expectation that nuclei having reflection asymmetric ("octupole") shapes do occur. A significant number of nuclides among the light isotopes of the elements Ra-Pa are now believed to be characterized by sizeable octupole deformations, in addition to those of even multipole order. (We refer the reader to Ref. [1], where several of the relevant references are given.) Among the more prominent features associated with octupole deformation are the existence of "parity-doublet" bands in the level schemes of odd-mass nuclides and, among the higher-spin yrast states in a number of the doubly even Ra and Th nuclei, a band of states of alternating parity connected by strongly enhanced El transitions.

As regards the evaluation of nuclear-structure data for these nuclides, most of the considerations mentioned above regarding the analysis of rotational-band structure still apply. The presence of the static quadrupole deformation leads to well developed rotational-band structures, which can still be analyzed in terms of the usual approaches. There are also a number of new considerations to be kept in mind. These include:

(i) The presence of parity-doublet bands in the low-energy spectrum of an odd-mass nuclide means that for each band of a given K-value there will be "nearby" another band with the same value of K, but of opposite parity. Since these bands represent projections into the laboratory frame from a single "intrinsic" state of mixed parity, a number of their properties are expected to be closely related. This knowledge may help the evaluator as he/she considers various possibilities for assigning Jπ values and grouping levels into rotational bands.

(ii) The presence of octupole deformation can significantly rearrange the expected ordering and energies of the one-quasiparticle states. Consequently, the spectrum of "Nilsson" states encountered in an odd-mass octupole-deformed nucleus (independent of the parity doubling) may be considerably different from that expected in the absence of octupole deformation.

(iii) It is difficult to associate a given one-quasiparticle state in these nuclides with a specific Nilsson orbital. This situation is rendered even more difficult by the fact that the quadrupole deformations of those nuclides thus far identified as being "octupole deformed" are generally smaller than for the rare-earth nuclides, so that the asymptotic quantum number labeling is even less "good" here than for the rare-earth nuclides. Thus, while the evaluators should feel free to derive and quote band parameters and K values as customary for bands in these nuclei, associating a specific Nilsson orbital (with the usual asymptotic quantum number labeling) with a given rotational band is more difficult to justify and, in the absence of compelling evidence to the contrary, should be avoided.

(iv) In the presence of reflection-asymmetric shapes, the "signature" symmetry is no longer valid. The only valid symmetry now is reflection in the nuclear yz-plane; and the associated quantum number is referred to [15] as the simplex, s. The Jπ values that occur in rotational bands characterized by the different values of the simplex are:

for s = +1, Jπ = 0+, 1-, 2+, 3-, etc.
for s = -1, Jπ = 0-, 1+, 2-, 3+, etc
for s = +i, Jπ = 1/2+, 3/2-, 5/2+, 7/2-, etc.
for s = -i, Jπ = 1/2-, 3/2+, 5/2-, 7/2+, etc.

Thus, for example, for the "octupole-deformed" doubly even isotopes of Ra and Th, the yrast (ground-state) bands above a certain J value contain alternating even-spin and odd-spin members, with even and odd parity, respectively. They would be assigned a value of the simplex, s, of +1. The simplex occupies the same position for the reflection-asymmetric nuclides as the signature does for the reflection-symmetric nuclides; and it is suggested that it be incorporated into nuclear-data evaluations in the same way as has been suggested above for the signature.

The nuclide ^{225}Ra, which is discussed in some detail in Sect. III below, is believed to be a good example of an "octupole-deformed" nucleus. Its level structure is significantly influenced by the octupole shape, although the evaluation considerations presented there are quite broad in their applicability and do not rely specifically for their validity on the existence of a stable octupole deformation. Octupole deformation is expected [16] to occur in other mass regions in addition to the light isotopes of Ra-Pa and, if such phenomena are indeed found there, then these ideas will have a much broader applicability than simply to this rather small portion of the Nuclide Chart.

C. Additional Considerations

In addition to the relative simplicity of the energy relationships among the members of a rotational band, the strongly deformed nuclei possess a number of other features that can significantly assist the evaluator of nuclear-structure data in making reliable Jπ assignments. Among these, we mention the following.

1. Occupation and relative alignment of Nilsson orbitals.

In contrast with the situation in the "spherical" nuclei, in strongly deformed nuclei each single-particle (or one-quasiparticle) "Nilsson" state can contain at most two (quasi)particles. In most situations of concern to the evaluator, the Jπ value of the band head of a given rotational band will be equal to the Kπ value of the intrinsic configuration upon which the band is built.

In considering the possible bands that can be formed from the coupling of two (or more) particles in a strongly deformed nucleus, it should be noted that the projections of the intrinsic spins (=1/2) of the particles on the nuclear symmetry axis can be either +1/2 or -1/2. Consequently the K values (and, hence, the band-head J value) for the states consisting of two particles in Nilsson orbitals having K values of K_1 and K_2 can have only the two possibilities K_1+K_2 and $|K_1$-$K_2|$. Further, as discussed by Gallagher and Moszkowski [17], of the two possible relative orientations of the single-particle orbitals occupied by the two odd particles in a doubly odd nucleus, the configuration resulting from the parallel coupling ($\Sigma = \Sigma_1 + \Sigma_2 = 1$) of the intrinsic spins of the odd particles should lie lower than that resulting from their antiparallel ($\Sigma = 0$) coupling. The only presently known exception to this rule in the doubly odd strongly deformed nuclei is ^{166}Ho, where the $K\pi = 0^-$ coupling of the two states $7/2^-[523]$p and $7/2^+[633]$n lies below their $K\pi = 7^-$ ($\Sigma = 1$) coupling. Even here, though, the energy separation of these two band heads is only ~6 keV, and a detailed treatment of the contribution of additional residual interactions can account for this shift.

Consequently, the evaluator can frequently rather severely restrict the number of $J\pi$ values to be considered for levels in deformed odd-odd nuclides using this "rule". While $J\pi$ assignments based solely on these considerations should probably not be regarded as being based on "strong" arguments, the evaluator can have confidence in adopting values based on such considerations, especially if additional evidence is available which helps support them.

For the two-quasiparticle states in the doubly even nuclei, it is expected that the band with the Σ=0 coupling of the two particle states will lie lower than that having $\Sigma = 1$ (see, e.g.,Ref. [3].) Although this appears to be correct in a number of well studied cases, the situation is less clear than in the odd-odd nuclides. This is due in large measure to the fact that these bands occur relatively high up (above the pairing gap, or $\gtrsim$1-1.5 MeV), where the density of states is rather high and it is often difficult to establish configurations for the bands and to identify both the Σ=0 and Σ=1 couplings of the two orbitals. A further complication in these cases is the occurrence of vibrational and other collective degrees of freedom in the general vicinity of these two-quasiparticle states; and this can significantly alter the energies of those bands whose $K\pi$ values are the same as those of these collective states.

For doubly odd nuclides, the residual neutron-proton interaction can give rise to an "odd-even" shift of the levels of K=0 bands, as discussed by Newby [18]. Special care should be exercised in dealing with such bands, especially in attempting to quote realistic values of the band parameters for them.

2. Allowed-Unhindered (au) Beta Transitions

Where present in a decay scheme, allowed-unhindered (au) β transitions[†] can be one of the most powerful tools available to an evaluator in deciding upon $J\pi$ and nucleonic-configuration assignments for nuclear states. The term "allowed-unhindered" denotes an allowed (i.e., $\Delta J = 0, \pm 1$ with no change in parity) β transition for which there is also no change in the asymptotic quantum numbers (i.e., N, n_z, Λ) between the initial and final states of the transforming nucleon. Two such orbital pairs are of importance in the rare-earth region, namely $7/2\text{-}[523]_p$ and $5/2\text{-}[523]_n$ in the lower-mass portion of this region and $9/2\text{-}[514]_p$ and $7/2\text{-}[514]_n$ in the upper-mass portion. No such orbital pairs are as yet observed to play a similar role among the strongly deformed actinide nuclides and, thus, au β decay is not yet an important process for the evaluator of these data.

The identifying characteristic of an au β transition is its small log ft value. It is not possible to establish a limit which uniquely separates **all** allowed-unhindered transitions from transitions that are not au. Certainly, all transitions having log ft values ≤5.0 can be considered to be au. In addition, a number of au transitions have log ft values as large as 5.2 or 5.3. However, there are also some instances where transitions that are not au have log ft values as small as ~5.2. Thus, some care is

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[†] Unless otherwise specified, the symbol β is used to denote both the β^- and the $\varepsilon+\beta^+$ decay process.

required in establishing whether or not β transitions whose log ft values are >5.0 but ≤5.5 are in fact au.

Once, however, it is established that a given β transition is indeed au, one is justified in concluding that one or the other of these orbital pairs is involved. This usually enables one to make quite well founded Jπ and configuration assignments for the initial and final states. With these established, it is frequently possible from them to establish Jπ assignments for other states as well.

Perhaps more interesting, however, is the situation in which one of the states involved in the β-decay process has a more complicated structure. Here, the observation of an au β transition makes it possible to convincingly establish the presence of such a structure. For example, in the β-decay of ^{163}Tb (Jπ = 3/2+, with a 3/2+[411] Nilsson-orbital assignment), a state at ~884 keV in ^{163}Dy is observed to be populated via a β- transition having log ft = 5.0. This transition is clearly au and thus must take place between 5/2-[523]n and 7/2-[523]p. As discussed in, e.g., Ref. [4], the only possibility for this is that the final state has Kπ = 1/2+, with (at least a sizeable component of) the three-quasiparticle configuration 3/2+[411]p + 5/2-[523]n - 7/2-[523]p. This three-quasiparticle configuration can be regarded as the two proton-quasiparticle configuration 3/2+[411]p - 7/2-[523]p coupled to the neutron state 5/2-[523]. Since this two-proton configuration is predicted [19] to be the dominant component of the Kπ = 2- octupole phonon in this region of the deformed nuclei, this Kπ = 1/2+ state can be interpreted [4] as being a Kπ = 2- octupole vibration built on the ^{163}Dy ground state, 5/2-[523]n. Since the 2- octupole-vibrational band is found to occur at relatively low energies in the doubly even nuclides in this mass region (at 1.148 MeV in ^{162}Dy [20], for example), the low value (884 keV) for the energy of such an excitation in ^{163}Dy is reasonable.

Note that, while the available data on this final state are certainly consistent with J,Kπ = ½,½+, it, together with the configuration assignment, could have been made solely on the basis of the existence of the au β- transition (and, of course, the ^{163}Tb ground-state Jπ value). Again, the question of whether such an assignment can be regarded as being based on a "strong" or on a "weak" argument can be debated. We feel, however, that an evaluator would be well justified in considering it to be "definitely" established.

3. Alpha Transitions†

The so-called "favored" α transitions (those for which the hindrance factor lies between 1 and, ~4 cf. Ref. [5]) take place between nuclear states having essentially identical configurations. Thus they make it possible to establish both Jπ values and nucleonic configuration assignments for a given final (initial) state if those of the initial (final) state are known. In the doubly even actinides, the favored transition is the ground state to ground state transition, and the transitions to the members of the ground-state rotational band are characterized by monotonically increasing, yet still relatively small, values of the hindrance factor.

In the odd-mass nuclei, the band head fed by the favored α transition need not be, and in most cases is not, the ground state. Again, however, the members of this band (the favored band) will be fed by α transitions having relatively small α hindrance factors, simplifying their identification. If the Jπ value of the parent-nuclide ground state is known, then those of the favored band are established as

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† Editor's note: See also **Appendix E Alpha-Decay Hindrance Factors**

well. Such a group of states in the daughter nucleus having "well established" Jπ values usually makes it possible to firmly establish Jπ values for many of the other observed states as well.

Other instances in which states may have rather low values of the α hindrance factor include β-vibrational (Kπ = 0+) states built on the favored band and, for "octupole-deformed" nuclei, the parity-doublet band associated with the favored band [21]. It should also be kept in mind that Coriolis mixing (See II.C.5. below) with members of the favored band may cause some states to have α-hindrance factors that are much smaller than would otherwise have been expected (for the unmixed state).

4. Intensity Relationships

An interesting aspect of the states in the strongly deformed nuclides is the existence of simple "geometrical" (Clebsch-Gordan-coefficient) relationships among the intensities of β and γ transitions between states that are members of rotational bands. While these so-called Alaga rules [22] are of considerable interest for nuclear-structure physics, these relationships are often obscured or modified by other effects to such an extent that their simple predictions are frequently not realized in actual nuclei. Since these confounding influences must be explicitly taken into account and since this is frequently a rather complicated and time-consuming task, the Alaga-rule relationships are not usually of much help to an evaluator. The β transitions, for example, frequently involve more than one angular-momentum value. Further, Coriolis mixing of a given band with one which is populated by an au β transition introduces an admixture which, through its inherently large β-decay matrix element, can significantly distort the predicted pattern of β feeding.

For γ-ray transitions, the relative intensities of the El transitions from a given one-quasiparticle state to various members of the rotational band built on another such state are known [4] to deviate significantly from the simple Alaga-rule predictions. On the other hand, collective El transitions appear to obey them quite well. Here, however, such transitions most commonly take place between octupole vibrational bands and their associated ground-state bands; and the strong Coriolis mixing between octupole-vibrational bands introduces strong changes in the observed γ-ray intensities which must be explicitly taken into account (see, e.g., Refs.[13] and [23]) before the simple underlying intensity patterns can be recovered.

Relative M1 transition probabilities from a given initial state to various members of a rotational band can frequently be well accounted for, but the contribution to the observed γ-ray intensities from the possible E2 admixtures need to be taken into account in interpreting such data.
Relative interband E2 transition probabilities are strongly dependent on Coriolis (or other) mixing, which may introduce the very large matrix elements associated with the rotational E2 transitions. The E2 transitions within a rotational band are generally well described by the Alaga rules. They can thus be used to calculate M1 admixtures in mixed intraband Ml+E2 transitions. While this information is of considerable interest for nuclear structure its use as a means of providing Jπ assignments is generally not great, since it is usually necessary to establish these quantities before carrying out this analysis.

The role of Alaga-rule considerations in making Jπ assignments thus seems to be rather limited. The evaluator should definitely exercise careful judgment in applying them to specific situations. Carefully applied, they can occasionally, perhaps frequently, provide corroboration of assignments arrived at from other considerations.

5. Rotation-Particle (Coriolis) Mixing

The influence of rotation-particle (Coriolis) coupling in the low-energy level structure of strongly deformed nuclides is frequently important in arriving at meaningful $J\pi$ and rotational-band assignments(see e.g.,[4]). While, in principle, a detailed Coriolis-mixing analysis should be carried out for any level scheme for which $J\pi$ assignments are being proposed, such a procedure is not practical for the mass-chain evaluator (and for most other nuclear physicists as well!). However, there are some simple qualitative considerations that frequently can be useful to the evaluator in interpreting level-scheme information.

The Coriolis interaction couples states having the same values of $J\pi$ and K-values that differ by 1 unit. The matrix element for this interaction can be written [4]

$$H_{K,K+1}(J)=H_{K+1,K}(J)=$$

$$(-\hbar^2/2\mathfrak{J})P_{K,K+1}A_{K,K+1}[(J - K)(J + K + 1)]^{\frac{1}{2}} \quad \text{for } K\neq\tfrac{1}{2} \qquad (4,a)$$

$$(-\hbar^2/2\mathfrak{J})P_{\frac{1}{2},\frac{1}{2}}\cdot A_{\frac{1}{2},\frac{1}{2}}\cdot(-1)^{J-\frac{1}{2}}(J + \tfrac{1}{2}) \quad \text{for } K=\tfrac{1}{2} \qquad (4,b)$$

(For interactions involving a K=0 band, an additional factor $[1+\delta_{K,0}]^{\frac{1}{2}} = \sqrt{2}$ needs to be included in eq. (4,a).)

Here, $\hbar^2/2\mathfrak{J}$ serves as a sort of rotational constant, giving an overall scale for the interaction, and is generally given a value equal, or close, to the rotational constant, A, of the rotating core. $P_{K,K+1}$ is a pairing factor, which is frequently not too different from unity. The strength of the Coriolis mixing is seen to be strongly J-dependent (as might be expected).

The dependence of the interaction strength on the nature of the nuclear states involved enters through the term $A_{K,K+1}$. Numerical tabulations of these matrix elements appropriate for different types of nuclear states have been published [4,5,24]; and these make it possible to carry out quantitative estimates of the effects of Coriolis mixing in many simple situations. However, there are a number of features of the Coriolis interaction that are frequently helpful in providing useful qualitative insights. First, the interaction strength depends strongly on the j-value of the spherical shell-model state ($i_{13/2}$, $h_{11/2}$, etc.) from which the Nilsson orbitals originate. Within a given j-shell, the $A_{K,K+1}$ varies approximately as $[(j-K)(j+K+1)]^{1/2}$. Consequently, within the so-called "unique-parity" states ($i_{13/2}$ neutrons and $h_{11/2}$ protons in the rare-earth region and $j_{15/2}$ neutrons in the actinide region) the Coriolis-mixing effects are expected, and observed, to be quite large, especially among the orbitals with the smaller K values, In terms of the asymptotic quantum numbers of these orbitals, the following selection rules indicate those bands for which this coupling is "unhindered":

$$\Delta N = 0; \quad \Delta K = \pm 1; \quad \Delta n_z = -\Delta\Lambda = \mp 1.$$

Among the "non-unique-parity" states, these rules are still quite useful, in that the largest intrinsic Coriolis matrix elements tend to occur between Nilsson states originating from the same spherical shell model state. However, in these cases, there is generally a considerable amount of deformation-dependent j-mixing, which diminishes this selectivity to some extent.

In the doubly even nuclides, the octupole-vibrational bands all contain sizeable components of the unique-parity orbitals in their two-quasiparticle makeup. Hence, they are expected [24], and indeed found, to possess large values of $A_{K,K+1}$.

In addition to the pronounced effects on the energy-level structure of rotational bands when the Coriolis-mixing matrix elements are large, such mixing, even when rather weak, can significantly affect certain level properties when the admixture carries with it a large matrix element for that process. Some examples of this are the following. In interpreting the intensities of β transitions feeding members of a given rotational band, it is tempting to ignore the possibility of small admixtures of other configurations; and this is justified if the transition probability to the admixed configuration is comparable in magnitude to that of the principal configuration. However, if one of the possible admixed configurations is connected to the β-decaying state via an allowed-unhindered matrix element, then it may have a pronounced effect on the β intensities and thus needs to be considered in order to understand the data.

Similarly, in analyzing α-transition intensities, Coriolis mixing of various states, especially if expected to be weak, can usually be neglected. However, if the mixing can introduce even small amounts of the "favored" band into the states under consideration, then it is important to take such mixing into account. In fact, the observation of "unexpectedly" small values of the alpha hindrance factors in bands where only large values are expected is frequently strong evidence for the presence of such mixing; and this may provide the evaluator with helpful information when considering Jπ and configuration assignments (as illustrated for ^{225}Ra in the following section).

Interband E2 transition probabilities, especially, can be greatly influenced by Coriolis mixing. When two bands are mixed by the Coriolis interaction the admixtures in each state give rise to the very large E2 matrix elements associated with the nuclear rotational motion. Since the intrinsic "single-particle" interband E2 transition probabilities are usually small, the observed E2 transition probabilities may be dominated by the contributions from the Coriolis-mixed configurations. (Since the intraband M1 transition probabilities do not show such a collective enhancement, Coriolis mixing, especially when weak, usually does not significantly affect them.)

Consequently, when analyzing nuclear-structure data on the strongly deformed nuclei, it is important to consider the effects of Coriolis mixing.

III An Example: Rotational Bands in ^{225}Ra

As an example of how some of the considerations presented in Sects. I and II can be applied to the analysis of a "real-life" case, we discuss some of the features of the rotational-band structure of ^{225}Ra. A portion of the low-energy level scheme of this nuclide is shown in Fig. 1. These data are taken from a recent study [1] of the α-decay of ^{229}Th.

A. The Kπ = 3/2+ Band at 149.8 keV.

The Jπ = 3/2+ and 5/2+ assignments for the 149.8- and 179.7- keV levels, respectively, appear well established [1]. From the spacing of these two levels we compute the value A = 5.97 keV, using the expression $E_J = AJ(J+1)$. Then, we calculate 221.5 keV as the expected position of the Jπ = 7/2+ member of this band. This is quite close to the position of a level at 220.5 keV (although it also is not too far from a level at 226.9 keV), and it is thus tempting to assign this level as the expected 7/2+ state (as has tentatively been done in Ref.[25]). From these two energy-level spacings, we use eq.(1)

to obtain the following values for the parameters A and B: A = 6.16 keV and B = -23.9 eV. From these, we calculate the position of the 9/2+ member of this band to be 267.2 keV, quite close to the energy of an observed level at 267.97 keV. Thus, from this analysis, it appears that we are dealing with a rather "well behaved" Kπ = 3/2+ band whose band members up to 9/2+ are identified and whose level energies are well fit using a simple two-parameter formula with parameter values of A = 6.16 keV and B = -23.9 eV.

However, there are problems associated with this simple picture; and for reasons they present, the authors of Ref.[1] have proposed a quite different set of Jπ assignments. Briefly, due to Coriolis mixing with the near-lying "favored" Kπ = 5/2+ band, the α hindrance factor to the 7/2+ member of this band should be rather small, in contrast to the observed value of ~270 to the 220-keV level. Also the γ-decay pattern of the nearby 226.9-keV state is not what one would expect for a J,Kπ = 7/2, 3/2+ state.

The nearest candidate for the 7/2+ band member (which, because of its expected small α hindrance factor, should be populated) is a level at 243.50 keV, which has an α hindrance factor of 14 and a γ-decay pattern that is quite consistent with Jπ = 7/2+. Such an assignment implies a quite different band structure than the "simple" one presented above. To see to what extent it is reasonable, the rotational-parameter analysis proceeds as follows. From the 5/2+ - 3/2+ and 7/2+ - 5/2+ level spacings, values of 5.97 keV and 9.11 key, respectively, are computed for the parameter A. This large difference indicates a rather distorted band.

In view of these quite different A values, it is not reasonable to "fit" the level energies by including a B term in the analysis. Rather, it appears more reasonable to try the terms A and A_3 in eq.(1). Doing this, we obtain A = 7.02 keV and A_3 = -174 eV. Using these, we calculate the energy of the 9/2+ band member to be 275.4 keV, not too far from an established level at 272.27 keV, whose γ-decay properties are not inconsistent with Jπ = 9/2+ (cf. Fig.1). Assuming that this latter state is the Jπ = 9/2+ member of the band, we can compute values for 3 parameters. Including a B term, we compute A = 7.13 keV, A_3 = -180 eV and B = -9.08 eV. Although we have no extensive "systematics" to help judge whether or not this value of A_3 is reasonable, we note that, since the Kπ = 3/2+ band can be directly Coriolis coupled to Kπ = 1/2+ bands and since these usually have nonzero decoupling parameters, it may be relatively large. The value inferred for B is now rather small and, thus, not unreasonable. [An objection to assigning the 272-keV state as the 9/2+ member of this band is the rather large value (~200) of the hindrance factor of the α transition feeding this state. Since the J,Kπ = 9/2, 3/2+ state should be Coriolis mixed with the 9/2+ member of the "favored" Kπ = 5/2+ band, whose bandhead lies at 236.7 keV, a considerably smaller value of this α hindrance factor is expected.]

The evaluator would be justified, in our opinion, in assigning the 243.50-keV level as the 7/2+ member of this Kπ = 3/2+ band and tentatively assigning the 272.27-keV state as the Jπ = 9/2+ band member. The band parameters given for the band should be A and A_3, with respective values of 7.02 keV and -174 eV (or -0.174 keV), with a comment that they were computed from the energies of the first three band members. Since the 9/2+ assignment can be regarded as only tentative, it would probably not be appropriate to list a value for B (although, since it is small, the evaluator can have some confidence that the listed band parameters are not unreasonable).

The foregoing analysis serves to illustrate a number of important points. First, the use of the most obvious, and simple, rotational-band energy-spacing considerations, without other information on the level properties, led to a picture of the structure of the 3/2+ band that appeared quite plausible, with "reasonable" values for the parameters A and B. Second, however, the consideration of additional information that was available led to a quite different picture of the structure of this rotational band. Had these additional data not been available, the evaluator could have quite reasonably been led to make "incorrect" Jπ and band assignments that would have been considered to have been based on "reasonable" considerations. Third, it should be emphasized that the question of which, if indeed either, of these two pictures of the rotational band structure of this 3/2+ band is correct is open at this time (although the "A,A_3 - approach" is definitely favored by the authors of Ref. [1]). As such, this situation serves again to illustrate the quandry which the evaluator faces when he or she attempts to resolve apparently discrepant data in order to arrive at the "correct" conclusion. However, here, as is so frequently the case in the strongly deformed nuclei, the relative simplicity of the angular momentum coupling scheme permits these questions to be considered at a deeper level of sophistication than would be possible in nuclides where these simplifying features did not occur.

B. The Kπ=1/2+ Ground-State Band

We now consider the ground-state band, which has Kπ = 1/2+. The spins of the states up through 9/2+ (see Fig.1) appear well established at this time [1]. The band structure departs markedly from a simple J(J+1) energy-level spacing pattern, indicating in this case a large, and positive, value for the decoupling parameter. For this distorted band structure, the following questions naturally arise: (1) what are the band parameters; and (2) are higher-spin members of this band excited in the ^{229}Th α decay and, if so, what are their energies?

Starting with the customary two-parameter expression for K=1/2 bands [cf. eq.(3)]

$$E(J,\tfrac{1}{2}) - E(\tfrac{1}{2},\tfrac{1}{2}) = A[J(J+1) + (-1)^{J+\frac{1}{2}} (J+\tfrac{1}{2})\, \underline{a}] - A(\tfrac{3}{4} - \underline{a}), \qquad (5)$$

and using the energies of the Jπ = 3/2+ and 5/2+ band members (namely, 42.75 and 25.41 keV), one computes 5.39 keV and +1.65, respectively, for A and $\underline{a}$. From these, the energies of the Jπ = 7/2+ and 9/2+ members of the band are calculated to be 125.5 keV and 93.8 keV, respectively. The agreement between these calculated values and the observed level energies of 111.57 and 100.5 keV, respectively, is not very good, particularly in view of the fact that the difference in the calculated 7/2+ and 9/2+ level energies is ~32 keV, while the observed separation is only ~11 keV. If, instead, one uses the observed 1/2+,7/2+ and 9/2+ level energies to determine values for A and $\underline{a}$, one obtains A = 5.28 keV and $\underline{a}$ = +1.23. With these parameter values, the energies of the 3/2+ and 5/2+ levels are calculated to be 35.3 keV and 29.2 keV, respectively, which is not very good agreement. (This is reflected, of course, in the significantly different value of the decoupling parameter from this calculation.)

It might be argued that the absolute differences between the calculated and observed level energies of the 3/2+ and 5/2+ states are really not all that large (only 7.4 and 3.8 keV, respectively) and consequently one should not worry about them. However, the spacing between these two states (a reflection of the contribution from the rotational energy) is poorly predicted (6 keV calculated vs. 17 keV observed), especially considering the low energies involved.

The use of the term BX^2 (the one customarily assumed to be next in importance in a K = 1/2 band) does not help the situation. For example, if one uses the energies of the 1/2+ through 7/2+ states to determine the parameters A, B and a, one obtains the following values:

States used in the fit	Deduced parameter values			States not included in the fit		
Jπ				Jπ	Energy (keV)	
	A(keV)	a	B(eV)		Calculated	Observed
1/2+, 3/2+, 5/2+, 7/2+	6.17	+1.38	-97.1	9/2+	55.9	100.5
1/2+, 3/2+, 5/2+, 9/2+	5.26	+1.70	+17.9	7/2+	127.7	111.6

On the other hand, if one uses the 1/2+, 3/2+, 5/2+ and 9/2+ states to determine the parameter values, a quite different set of values is obtained, as shown above. Not only are these sets "unacceptably" different but also, rather than obtaining a better fit, the fit is considerably worsened. Furthermore, it can be shown that including a CX^3 term in the analysis does not really "solve" the problem. In this case, the four parameters can fit the four energy spacings exactly, but the resulting parameters, namely B = +280 eV and C = -8 eV, are so large that they can be regarded as being physically unreasonable.

It is interesting to note that the magnitude of the 7/2+ - 9/2+ level spacing is **smaller** than that of the 3/2+ and 5/2+ states. This situation cannot be reproduced, for any choice of parameters in the simple two-parameter formula (eq.(5)). If one wants to account for this fact, **phenomenologically** at least, another term must be considered which, like the decoupling parameter, has an alternating dependence on level spin. The B_1 term, viz.

$$(-1)^{J+\frac{1}{2}}(J+\tfrac{1}{2})B_1X,$$

is the logical choice for consideration. If one includes it and excludes the BX^2 term (that is, one uses A, a and B_1, one finds a good fit to the 1/2+ to 9/2+ level energies. Using, for example, the 1/2+ to 7/2+ level energies to determine the three parameters and then computing the energy of the 9/2+ state, one finds that the predicted 9/2+ energy is 104.8 keV, vs. the 100.5 keV observed. With the recognition of the importance of a "B_1 term" in the analysis, we can proceed to use the energies of the 1/2+ to 9/2+ states to determine values for the four parameters A, a, B, and B_1. The values obtained are as follows:

A = 5.11 keV; a = +1.89; B = -8.5 eV and B_1 = -178 eV.

Note that, now, the deduced value of B is much smaller (and more reasonable) than before and also that the decoupling parameter is different. Of course, the energies of the 1/2+ through 9/2+ states are now fit exactly. The resulting four-parameter rotational energy-level formula predicts the energies of some of the higher-spin members of this band to be 11/2+, 197.6 keV; 13/2+, 227.7 keV; and 15/2+, 283.8 keV. While the calculation of the higher levels with these parameter values may not be justified, it might be hoped that this prediction is good enough to be useful. There is as yet no evidence for a state near 197.6 keV that can be identified as 11/2+; and, although there is a state at 284.4 keV (not shown in Fig.1), near the expected position of the 15/2+ state, its decay properties [1] clearly indicate that it does not have Jπ=15/2+. There is, however, a state at 226.9 keV which is an excellent candidate for the Jπ = 13/2+ band member. Its γ-decay pattern (only one de-exciting γ ray, to the 9/2+ state) is just what one would expect for a 13/2+ state. Consequently, we feel justified in making a Jπ assignment of 13/2+ to the 226.93 keV state. Whether this Jπ = 13/2+ assignment should be regarded as being based on "strong", or on "weak", considerations is, perhaps, a matter of taste but, in the view of the authors of Ref. [1], a prudent evaluator would be well justified in making it. Note, in particular, that in regions of the Nuclide Chart where strongly deformed nuclear shapes do not occur, and eq.(1) is thus not applicable, there would have been essentially no real basis for concluding that the 226.9-keV state had Jπ = 13/2+.

C The Kπ = 1/2- Band

The negative-parity states below 130 keV in ^{225}Ra (cf. Fig.1) can be interpreted quite readily as members of a Kπ = 1/2- band, built on the Jπ = 1/2- state at 55.13 keV. Here, in contrast with the 1/2+ band, values deduced for the various parameters in the rotational energy-level expression are much less dependent on which levels are chosen to determine them. A "B_1 term" is found to be necessary here also. With the energies of the 1/2- to 7/2- levels used to determine values for the parameters A, $\underline{a}$ and B_1, the calculated energy for the 9/2- member of the band is 220.8 keV. This is quite close to the energy of an observed level at 220.5 keV. The γ-decay properties of this state agree quite well with those expected for a Jπ = 9/2- state. [As discussed in Sect. III. A. above, this level had been tentatively assigned in some studies as the 7/2+ member of a Kπ = 3/2+ band, but such an assignment is most likely incorrect.]

With the 220.51-keV level thus identified as the Jπ = 9/2- member of this Kπ = 1/2- band, one can use these four energy-level spacings to determine values for the four band parameters A, $\underline{a}$, B and B_1. The values thus obtained are

$$A = 5.11 \text{ keV}, \underline{a} = -2.56, B = -3.8 \text{ eV and } B_1 = +64.3 \text{ eV}.$$

From these, the following energies are calculated for the Jπ=11/2- to 15/2- states: 11/2-, 151.1; 13/2-, 348.2; and 15/2-, 276.3. Although no evidence for any of these states is (not unexpectedly) reported in the γ-decay study of Ref.[1], it is proposed from a recent (τ,α)-reaction study [26], that the 15/2- state occurs at 274 keV, in excellent agreement (especially in light of the experimental uncertainties) with the calculated value of 276 keV. This lends some support to the results of the rotational-band analysis. It also, perhaps, strengthens the evaluator's confidence in assigning Jπ = 9/2- to the 220.5-keV level. In our opinion, an evaluator would be well justified in making such an assignment to the 220.5-keV level and in listing as parameters for the Kπ = ½ band the four values given above.

D. The Kπ = 5/2+, "Favored" Band

The Kπ = 5/2+ band at 236.7 keV is the "favored" band in the α decay of ^{229}Th and, as such, is probably the most firmly established band in ^{225}Ra. Because of the small values of the hindrance factors of the α transitions feeding the 236.7- and 267.9-keV levels, we feel that the Jπ assignments of 5/2+[†] and 7/2+, respectively, are "certain" and that the 9/2+ and 11/2+ assignments, respectively,

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[†] The ground state of ^{229}Th has Jπ = $5/2^+$, with the most probable Nilsson-orbital assignment being $5/2^+$[633].

to the 321.8- and the 390.3-keV levels are well established.

The determination of realistic values of the rotational band parameters for this band, however, presents difficulties. There is a considerable amount of "staggering" within this band, as evidenced by the fact that the 7/2+ - 5/2+ and 9/2+ - 7/2+ energy differences give rise to predicted values for *A* of 4.47 keV and 5.98 keV, respectively. If one, recognizing this, uses a two-parameter form of eq.(1), including the parameters A and A_5, to describe the band, one obtains from the energies of the 5/2+ - 9/2+ states the values A = 4.85 keV and A_5 = -3.14 eV. Although these cannot be regarded as being unreasonable (note, in particular, that the magnitude of A_5 is considerably smaller than that deduced for A_3 in III. A above), they predict a value of 346.1 keV for the energy of the 11/2+ member of the band, whereas the observed energy of this state is 390.3 keV. If one ignores for the moment the need for an A_5 term (or some term with an alternating dependence on J) and uses only A and B parameters to describe this band, the energies of the 5/2+ - 9/2+ states yield values of 3.34 keV and +94.1 eV, respectively, for *A* and *B*. This value of *B* is regarded as being unreasonably large, on the grounds that the resulting value of the term BX^2 is not very much smaller than that of *AX*. For example, for $E_{7/2}$ - $E_{5/2}$, it is roughly one-third the size of *AX* and, for $E_{9/2}$ - $E_{5/2}$, it is ~60% as large. For the use of eq.(1) (with only a small number of parameters) to be justified, the contribution of the *B* (and successively higher order) terms must be much smaller than that of the *A* term.

One can, of course, use all three of these parameters and fit the energies of the 5/2+ through the 11/2+ band members exactly. This yields the following values: *A* = 3.78 keV, *B* = +66.9 eV and A_5 = -0.91 eV. Again, the value of *B* appears unreasonably large, and, of course, with these three values the ability of eq.(1) to predict the energies of higher-spin members of this band is highly questionable.

Thus, while the Jπ assignments of the first four members of this band appear quite well established, the energy relationship among the band members cannot be described using reasonable values of the rotational parameters. This may reflect the presence of strong Coriolis mixing of this bands with other positive-parity bands in ^{225}Ra. To explore this possibility in detail, however, lies outside the usual scope of an A-chain evaluation. Consequently, it is recommended that the evaluator simply point this out and not attempt to "adopt" any rotational-parameter values for this band.

E. Concluding Remarks

In the preceding discussion in this Section, we have illustrated some of the strengths and potential pitfalls in using eq.(1) to analyze the energy-level structure of rotational bands (at low rotational frequencies) in the strongly deformed nuclei. This approach has led [1] to a proposed picture of the low-energy rotational band structure of ^{225}Ra that differs considerably from that available previously. It has not led to any new proposals regarding the Kπ = 5/2+ band, but it has served to point out that a more detailed analysis of this band and its couplings to other bands is needed before any conclusions can be drawn regarding the origin of the problems encountered in trying to obtain reasonable values of its band parameters.

It should be noted, in passing, that the evaluation procedure described in this Section has not explicitly relied for its validity on the correctness of the assumption that ^{225}Ra is an "octupole-deformed" nucleus. However, the experimental evidence thus far available on ^{225}Ra is consistent with this hypothesis. The fact that the ground-state band has Kπ = 1/2+, for example, is strongly suggestive of a stable octupole deformation, since, otherwise, the lowest 1/2+ band in ^{225}Ra is expected to occur rather high up in the level scheme (~0.8 MeV). Similarly, the low energy of the

Kπ = 1/2- band finds a natural explanation as the parity-doublet partner of the 1/2+ band. The values of the decoupling parameters of these two bands are, as expected for a parity doublet, comparable in magnitude but opposite in sign. Further, they are quite different from the values that would be expected for any of the reflection-symmetric K = 1/2 bands. In this regard, the use of a "B_1-term" in the analysis has led to "better" estimates of these two decoupling-parameter values, especially for the 1/2+ band, than would have been obtained by neglecting it; and these two values are considerably closer together than the earlier estimates. However, the octupole-deformed coupling scheme has explicitly affected the conclusions drawn from the analysis, in that no "Nilsson" orbital assignments have been proposed for any of the bands.

In any event, it is hoped that this discussion will be helpful to the mass-chain evaluator in using these ideas as one potentially powerful tool for choosing among alternative Jπ values in the evaluation of complicated energy-level schemes in the strongly deformed nuclides.

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Appendix

Summary

The level schemes of strongly deformed nuclei possess a number of features that can materially assist the evaluator in making Jπ and nucleonic-configuration assignments. The existence of well developed rotational bands, with their inherently simple relationship between level energy and spin, the extensive systematics and relatively simple make-up of the intrinsic states upon which these bands are built, and a number of simplifying features of the angular-momentum coupling scheme that occur because of the existence of the deformation all combine to permit the knowledgeable evaluator to deduce quite reliable Jπ assignments from data sufficiently meager that one could draw almost no conclusions from them if the nuclide for which they were available was not deformed.

It is difficult to frame a compact set of rules for Jπ assignments that can be applied without exception in these situations. However, it is possible to lay out general considerations to assist the evaluator in the task of arriving at reliable Jπ assignments for levels in the strongly deformed nuclides. Below, we summarize some of these. Those features of the nuclear structure of the strongly deformed nuclei upon which they are based are discussed in the earlier sections of this document.

A. Level Energies and Quantum Numbers

The following expression is recommended for use in describing the level energies within a rotational band at low rotational frequencies:

$$\begin{array}{ll}
E(J,K)=E_{\mathrm{K}} + AX + BX^2 + \text{etc.} & \\
+(-1)^{\mathrm{J}+½}\,(J+1/2)(A_1 + B_1X + \text{etc.}) & \text{for K} = 1/2 \\
+(-1)^{\mathrm{J}+1}\,J(J+1)(A_2 + B_2X + \text{etc.}) & \text{for K} = 1 \\
+(-1)^{\mathrm{J}+3/2}\,(J-1/2)(J+1/2)(J+3/2)(A_3 + B_3X + \text{etc.}) & \text{for K} = 3/2 \\
+(-1)^{\mathrm{J}}\,(J - 1)J(J+1)(J + 2)(A_4 + B_4X + \text{etc.}) & \text{for K} = 2
\end{array}$$

where $X=J(J+1)-K^2$

For K = 1/2 bands, the decoupling parameter, $\underline{a}$, is related to the parameter A_1 through the expression $A_1 = A\underline{a}$.

For "well-behaved" rotational bands, the coefficients B and C are expected to be small, of the order of magnitude $B/A \sim 10^{-3}$ and $C/B \sim 10^{-3}$. Typical values for the rotational constant A, are ~12 keV in the rare-earth region and ~6 keV in the actinide region, although sizeable departures from these are observed. Other than that they are of the order of magnitude of unity, no general statement can be made regarding "typical" values of the decoupling parameter. They depend strongly on the configuration of the K = 1/2 band under consideration; and, in fact, knowledge of the decoupling parameter gives a good insight into the configuration assignment of the band. The parameters $A_{2\mathrm{K}}$ are expected to decrease rapidly with increasing K-value, but no extensive systematics of such values is available at present. Evaluators should be encouraged to give more attention in their analyses of rotational-band structure to the influence of $A_{2\mathrm{K}}$-type terms, especially for those bands having smaller values of K (say, 5/2 or less), where the influence of such terms is more pronounced.

In the analysis of "high-spin" states (those generally accessible only to in-beam spectroscopy or heavy-ion-induced Coulomb excitation), use of the rotational energy-level formula above to deduce band parameters is generally not justified and can lead to "unphysical" conclusions. At the high rotational frequencies associated with such states, the appropriate quantum number is no longer K but rather the "signature", α (together with the parity). The following relations exist between α and the total angular momentum, J:

α = 0, J = 0, 2, 4, etc.,
α = 1, J = 1, 3, 5, etc.,
α = +1/2, J = 1/2, 5/2, 9/2, etc.,
α = -1/2, J = 3/2, 7/2, 11/2 etc.,

For nuclides that are believed to have reflection-asymmetric ("octupole-deformed") shapes, the quantum number associated with the appropriate nuclear symmetry is, instead of the signature, the "simplex", $\underline{s}$. The Jπ values that occur in rotational bands characterized by the different values of the simplex are:

s = 0, Jπ = 0+, 1-, 2+, 3-, etc,
s = 1, Jπ = 0-, 1+, 2-, 3+, etc.
s = +i, Jπ = 1/2+, 3/2-, 5/2+, 7/2-, etc.,
s = -i, Jπ = 1/2-, 3/2+, 5/2-, 7/2+, etc.

B. Gallagher-Moszkowski Rules

In predicting the relative ordering of the two configurations resulting from the parallel ($\Sigma = 1$) and antiparallel ($\Sigma = 0$) coupling of the intrinsic-spin projections of the two odd (quasi)particles in doubly odd deformed nuclides, the Gallagher-Moszkowski rules indicate that the $\Sigma = 1$ coupling should lie lower. In doubly even nuclides, the opposite should be the case. In the doubly odd nuclides, only one exception to these "rules" is presently known. Consequently, in analyzing the level structure of these nuclides, the evaluator can, with some degree of confidence assume, in the absence of other information, that the $\Sigma = 1$ coupling will lie lower. There are, however, a number of complicating factors in the level structure of doubly even nuclei which, in the absence of other information, make these considerations of relatively limited use in the evaluation of data on these nuclides.

C. Allowed-Unhindered Beta Transitions

Allowed-unhindered (au) p transitions are ones in which the asymptotic quantum numbers of the initial-and final-state orbitals of the transforming nucleon are the same. Their systematic occurrence thus far appears confined to the "rare-earth" region and uniquely establishes the presence of either the orbital pair 7/2-[523]p, 5/2-[523]n or 9/2-[514]p, 7/2-[514]n. With this knowledge, and the relatively simple angular-momentum coupling rules that apply, it is usually possible not only to make definitive Jπ assignments but also to provide entirely reliable configuration assignments to the states involved. Beta transitions in this region having log ft values of 5.0 or less can be confidently assigned as being au. In addition, many au transitions are observed which have log ft values as large as ~5.5. However, as regards Jπ and configuration assignments, some caution must be exercised in classifying as au newly encountered β transitions whose log ft values lie between ~5.0 and ~5.5, since a few cases are known were β transitions with log ft values as low as ~5.2 do not take place between one or the other of these two orbital pairs.

D Favored Alpha Transitions

"Favored" α transitions involve no change in nucleonic configuration between the initial and final states. In the doubly even nuclides such transitions take place between the ground states of the parent and daughter nucleus, while in the odd-A and doubly odd nuclides the final state is generally an excited state. The characteristic feature of favored α transitions in these latter two categories of nuclei is an α hindrance factor in the range from unity to ~4. The observation of a favored α transition is thus a strong basis for making both Jπ and nucleonic-configuration assignments. Further, the members of the so-called "favored" band (the band built on the state fed by the favored transition) are fed by α transitions whose hindrance factors, although increasing monotonically with final-state spin, are nonetheless still relatively small and hence usually readily identifiable. In analyzing α-hindrance-factor information to provide Jπ and nucleonic-configuration assignments, however, it needs to be kept in mind that other phenomena can also give rise to small hindrance factors. These include Coriolis mixing with the favored band, the presence of octupole deformation and β vibrational excitations built on the favored band.

E. Alaga-Rule Considerations

The Alaga rules, which relate the relative values of the reduced transition probabilities of various decay processes from an initial state to various final states that are members of the same rotational band, are usually of little use to the evaluator in arriving at Jπ assignments. This results primarily because the essential simplicity of the ideas underlying them is frequently masked by other effects which are difficult to take explicit account of. Consequently, their usefulness usually lies in providing corroboration to assignments proposed from other considerations. The intraband E2 transitions represent one exception to this statement. It is found that the reduced transition probabilities of these transitions are well described by the Alaga-rule predictions; and the evaluator can use this observation to infer E2/M1 mixing ratios for intraband cascade transitions (for which $\Delta J = 1$) when both the cascade γ ray and its corresponding crossover ($\Delta J = 2$) γ ray are observed. Another potential exception may be the El transition probabilities when collective effects (e.g., octupole vibrations, reflection asymmetry) are important. Careful attention should be given to the analysis of such situations, but an emerging body of evidence suggests that one can use the Alaga rules to draw correct conclusions in such situations.

F. Rotation-Particle (Coriolis) Coupling

In evaluating nuclear structure data for strongly deformed nuclei, it is important to keep in mind that rotation-particle (Coriolis) coupling may have a significant effect on certain level properties. Although a proper analysis of such effects requires calculations utilizing large computer-based codes, there are simple qualitative considerations which can frequently provide sufficient insight to permit the evaluator to draw meaningful conclusions from the data without the necessity of such calculations. The Coriolis interaction mixes states having the same Jπ values and K values that differ by one unit, and this mixing increases with decreasing separation of the states. It is strongest among states that originate from the same spherical shell-model state, and increases with increasing j value (and for a given j, decreasing K value). It is thus especially strong among the so-called "unique-parity" states (i.e., $i_{13/2}$ neutrons and $h_{11/2}$ protons in the rare-earth region and $j_{15/2}$ neutrons in the actinides). In these cases the selection rules in the asymptotic quantum numbers for "unhindered" Coriolis coupling are $\Delta N = 0$, $An_z = -\Delta\Lambda = \pm 1$. In addition to the large distortions that are produced in the level structure of rotational bands through strong Coriolis mixing, even weak mixing can produce pronounced effects on various nuclear properties when the mixing brings in a large matrix element for the associated process. Examples of these include α-hindrance factors, β-decay log ft values and B(E2) values when the mixing introduces the unusually large matrix elements associated with favored α decay, au β transitions and intraband E2 transitions, respectively.