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Table of Nuclear Root Mean Square Charge Radii

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Summary description by
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Abstract: This document describes a table of nuclear root-mean-square (*rms*) charge radii evaluated by two different procedures. The data are available from the IAEA Nuclear Data Section via INTERNET or on PC diskettes upon request. This document supersedes the previous IAEA-NDS-163, 1990, “Nuclear Charge Radii”.

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I. Angeli, Table of Nuclear Root Mean Square Charge Radii,
Report INDC-(HUN)-033 (1999).

SUMMARY

Nuclear root-mean-square (***rms***) charge radii have been compiled, selected and evaluated using two different averaging procedures: a refined (**RFOR**) and a simpler (**REXC**). The same input data (fast electron scattering, muonic atom x-rays and optical isotope shifs) were used for both procedures, the results for weighted average radii are very close to each other: 91% of the differences are within ±1/2 of the combined uncertainties. This confirms that ***rms*** charge radii are less sensitive to the averaging procedure than they are to the selection of the input data.

The resulting nuclear ***rms*** charge radii –as of May 1999- are presented in tabular form for the two procedures in the following table.

The data and more details on the evaluation procedure are available on the NDS/IAEA web page: http://www-nds.iaea.org/ and on PC diskettes free of charge upon request.

References:

1. I.Angeli: Table of Nuclear Root Mean Square Charge Radii, INDC-(HUN)-033; Nuclear Data Section, IAEA, (1999).
2. I.Angeli: Acta Phys. Hung. **69** (1991) 233
3. I.Angeli: Proc. 6th Int. Conf on Nuclei far from Stab., 9th Int. Conf. on Atomic Masses and Fundamental Constants, (19-24 July 1992, Bernkastel-Kues, Germany), Inst. Phys. Conf. Ser. No. 132: Sect. 1: p.89
4. I.Angeli: Acta Phys. Hung. New Series, Heavy Ion Physics, **8** (1998) 23

Diskette 1.

File name	*Contents*
0 Readme.doc	Informative text
1 Maintext.doc	Description of evaluation procedures
2 Figure1.doc	Figure 1. of the main text

***Diskette 2*.**

File name	*Contents*
App.I._Radius_references_(diagram).doc	Diagram with statistics for references
App.II._Input_radius_data.doc	Input data
App.III._FORTRAN_program.doc	FORTRAN program
App.IV._Average_radii.doc	Table of *rms* charge radii (this table)
App.V._Refs_for_elscatt.doc	References
App.VI._Refs_for_mu_X-rays.doc	References

Table of Nuclear Mean Square Charge Radii

Results obtained by a refined (R_{FOR}) and a simple (R_{EXC}) evaluation procedure.
Updated: 1 May 1999

Element	Z	A	R_{FOR} [fm]*	ΔR_{FOR} [fm]*	R_{EXC} [fm]*	ΔR_{EXC} [fm]*	$\frac{R_{FOR}-R_{EXC}}{\Delta(R_{FOR}-R_{EXC})}$
n	0	1	-0.1149	.0024	-0.1149	.0027	.01
H	1	1	0.8521	.0069	0.8520	.0070	.01
		2	2.1300	.0120	2.1300	.0120	.00
		3	1.7591	.0356	1.7591	.0363	.00
He	2	3	1.9373	.0296	1.9408	.0240	-.09
		4	1.6758	.0026	1.6757	.0028	.04
Li	3	6	2.5521	.0311	2.5522	.0333	.00
		7	2.3952	.0506	2.3953	.0514	.00
Be	4	9	2.5180	.0114	2.5180	.0119	.00
B	5	10	2.4278	.0492	2.4277	.0499	.00
		11	2.4059	.0291	2.4060	.0294	.00
C	6	12	2.4704	.0023	2.4705	.0023	-.02
		13	2.4625	.0036	2.4625	.0037	.00
		14	2.4978	.0126	2.4966	.0165	.06
N	7	14	2.5519	.0083	2.5520	.0087	-.01
		15	2.6094	.0085	2.6095	.0094	-.01
O	8	16	2.7061	.0084	2.6995	.0068	.61
		17	2.6975	.0136	2.6980	.0173	-.02
		18	2.7603	.0141	2.7599	.0162	.02
F	9	19	2.8976	.0027	2.8976	.0028	.01
Ne	10	20	3.0046	.0038	3.0045	.0038	.01
		21	2.9670	.0036	2.9670	.0036	.00
		22	2.9544	.0035	2.9544	.0035	-.01
Na	11	23	2.9935	.0024	2.9935	.0024	-.01
Mg	12	24	3.0575	.0024	3.0574	.0026	.01
		25	3.0269	.0034	3.0278	.0042	-.17
		26	2.9987	.0328	3.0135	.0232	-.37
Al	13	27	3.0527	.0078	3.0566	.0058	-.41
Si	14	28	3.1234	.0027	3.1236	.0030	-.04
		29	3.1131	.0075	3.1140	.0086	-.08
		30	3.1595	.0163	3.1494	.0132	.48
P	15	31	3.1893	.0019	3.1893	.0021	.01
S	16	32	3.2522	.0059	3.2572	.0045	-.68
		34	3.2848	.0030	3.2848	.0030	.00
		36	3.2984	.0031	3.2987	.0041	-.07
Cl	17	35	3.3652	.0145	3.3654	.0191	-.01
		37	3.3840	.0170	3.3840	.0170	.00
Ar	18	36	3.3910	.0058	3.3917	.0053	-.09
		38	3.4035	.0031	3.4035	.0036	.01
		40	3.4273	.0027	3.4273	.0029	.00

El	Z	A	R_{FOR} [fm]*	ΔR_{FOR}	R_{EXC} [fm]*	ΔR_{EXC}	$\frac{(R_{FOR}-R_{EXC})}{\Delta(R_{FOR}-R_{EXC})}$
K	19	39	3.4350	.0020	3.4350	.0022	.01
		41	3.4520	.0033	3.4520	.0033	.00
Ca	20	40	3.4766	.0010	3.4766	.0010	-.01
		42	3.5057	.0013	3.5057	.0013	.00
		43	3.4928	.0011	3.4928	.0011	.00
		44	3.5152	.0013	3.5152	.0013	.00
		46	3.4921	.0012	3.4921	.0012	.00
		48	3.4737	.0011	3.4737	.0011	-.01
Sc	21	45	3.5460	.0035	3.5460	.0035	.00
Ti	22	46	3.6060	.0032	3.6060	.0032	.00
		47	3.5960	.0032	3.5960	.0032	.00
		48	3.5916	.0031	3.5916	.0032	-.01
		49	3.5740	.0030	3.5740	.0030	.00
		50	3.5710	.0031	3.5710	.0031	.00
V	23	51	3.6000	.0030	3.6000	.0030	.00
Cr	24	50	3.6595	.0033	3.6597	.0044	-.03
		52	3.6452	.0042	3.6452	.0044	.01
		53	3.6590	.0030	3.6590	.0030	.00
		54	3.6854	.0031	3.6854	.0031	.01
Mn	25	55	3.7060	.0033	3.7060	.0033	.00
Fe	26	54	3.6929	.0048	3.6929	.0049	.00
		56	3.7383	.0030	3.7383	.0031	-.01
		57	3.7540	.0037	3.7540	.0037	.00
		58	3.7735	.0037	3.7735	.0037	-.01
Co	27	59	3.7883	.0028	3.7883	.0029	-.01
Ni	28	58	3.7745	.0020	3.7745	.0022	.01
		60	3.8116	.0028	3.8117	.0029	-.03
		61	3.8211	.0034	3.8212	.0043	-.03
		62	3.8387	.0035	3.8390	.0041	-.06
		64	3.8584	.0033	3.8584	.0040	-.01
Cu	29	63	3.8877	.0053	3.8853	.0049	.33
		65	3.9029	.0028	3.9029	.0028	-.01
Zn	30	64	3.9277	.0033	3.9277	.0034	.01
		66	3.9488	.0028	3.9489	.0030	-.02
		68	3.9657	.0028	3.9657	.0029	-.01
		70	3.9833	.0032	3.9833	.0033	.01
Ga	31	69	3.9960	.0032	3.9960	.0032	.00
		71	4.0110	.0032	4.0110	.0032	.00
Ge	32	70	4.0419	.0017	4.0419	.0017	-.01
		72	4.0586	.0017	4.0587	.0020	-.02
		73	4.0610	.0033	4.0610	.0033	.00
		74	4.0743	.0017	4.0743	.0017	.02
		76	4.0808	.0017	4.0808	.0017	.02
As	33	75	4.0960	.0033	4.0960	.0033	.00
Se	34	74	4.0700	.0200	4.0700	.0200	.00
		76	4.1426	.0045	4.1416	.0057	.13

El	Z	A	R_{FOR} [fm]*	ΔR_{FOR}	R_{EXC} [fm]*	ΔR_{EXC}	$(R_{FOR}-R_{EXC})$ / $\Delta(R_{FOR}-R_{EXC})$
Se	(contd.)	77	4.1390	.0035	4.1390	.0035	.00
		78	4.1399	.0034	4.1399	.0035	.00
		80	4.1409	.0023	4.1409	.0025	-.01
		82	4.1376	.0039	4.1380	.0051	-.06
Br	35	79	4.1630	.0035	4.1630	.0035	.00
		81	4.1600	.0034	4.1600	.0034	.00
Kr	36	78	4.2032	.0016	4.2032	.0016	.00
		80	4.1976	.0013	4.1976	.0013	.00
		82	4.1921	.0015	4.1921	.0015	.00
		83	4.1860	.0018	4.1860	.0018	.00
		84	4.1884	.0016	4.1884	.0016	.00
		86	4.1839	.0017	4.1839	.0017	.00
Rb	37	85	4.2040	.0036	4.2040	.0036	.00
		87	4.1990	.0036	4.1990	.0036	.00
Sr	38	84	4.2365	.0017	4.2365	.0017	.00
		86	4.2261	.0013	4.2261	.0013	.00
		87	4.2190	.0012	4.2190	.0012	.00
		88	4.2090	.0068	4.2171	.0050	-.96
Y	39	89	4.2438	.0032	4.2439	.0032	-.02
Zr	40	90	4.2692	.0014	4.2692	.0014	.00
		91	4.2850	.0017	4.2850	.0018	.00
		92	4.3055	.0014	4.3055	.0014	.00
		94	4.3314	.0015	4.3314	.0015	.00
		96	4.3508	.0016	4.3508	.0016	.00
Nb	41	93	4.3248	.0025	4.3248	.0027	.00
Mo	42	92	4.3146	.0015	4.3146	.0015	.00
		94	4.3518	.0014	4.3518	.0014	.00
		95	4.3617	.0013	4.3617	.0013	.00
		96	4.3840	.0011	4.3840	.0011	.00
		97	4.3880	.0011	4.3880	.0011	.00
		98	4.4089	.0014	4.4089	.0014	.00
		100	4.4465	.0018	4.4465	.0018	.00
Ru	44	96	4.3930	.0047	4.3930	.0047	.00
		98	4.4230	.0055	4.4230	.0055	.00
		99	4.4350	.0042	4.4350	.0042	.00
		100	4.4530	.0043	4.4530	.0043	.00
		101	4.4610	.0042	4.4610	.0042	.00
		102	4.4810	.0042	4.4810	.0042	.00
		104	4.5090	.0043	4.5090	.0043	.00
Rh	45	103	4.4940	.0041	4.4940	.0041	.00
Pd	46	102	4.4840	.0044	4.4840	.0044	.00
		104	4.4780	.0269	4.4964	.0196	-.55
		105	4.5160	.0046	4.5160	.0046	.00
		106	4.5061	.0221	4.5227	.0164	-.60
		108	4.5462	.0091	4.5506	.0090	-.35
		110	4.5759	.0132	4.5739	.0102	.12

El	Z	A	R_{FOR} [fm]*	ΔR_{FOR}	R_{EXC} [fm]*	ΔR_{EXC}	$\frac{(R_{FOR}-R_{EXC})}{\Delta(R_{FOR}-R_{EXC})}$
Ag	47	107	4.5440	.0042	4.5440	.0042	.00
		109	4.5650	.0042	4.5650	.0042	.00
Cd	48	106	4.5340	.0045	4.5340	.0045	.00
		108	4.5540	.0045	4.5540	.0045	.00
		110	4.5752	.0037	4.5752	.0038	.01
		111	4.5790	.0044	4.5790	.0044	.00
		112	4.5974	.0041	4.5971	.0054	.04
		113	4.5970	.0043	4.5970	.0043	.00
		114	4.6140	.0039	4.6136	.0041	.07
		116	4.6282	.0039	4.6281	.0049	.01
In	49	113	4.5980	.0050	4.5980	.0053	.00
		115	4.6189	.0052	4.6177	.0058	.15
Sn	50	110	4.5807	.0064	4.5807	.0064	.00
		111	4.5859	.0061	4.5859	.0061	.00
		112	4.5915	.0038	4.5915	.0048	.00
		113	4.6038	.0047	4.6038	.0047	.00
		114	4.6082	.0033	4.6082	.0042	.00
		115	4.6167	.0038	4.6167	.0038	.00
		116	4.6278	.0030	4.6278	.0030	.00
		117	4.6310	.0023	4.6310	.0027	.00
		118	4.6411	.0018	4.6412	.0023	-.02
		119	4.6456	.0015	4.6456	.0018	-.01
		120	4.6544	.0011	4.6545	.0014	-.03
		121	4.6589	.0013	4.6589	.0013	.00
		122	4.6648	.0017	4.6648	.0021	-.01
		123	4.6684	.0020	4.6684	.0020	.00
		124	4.6752	.0025	4.6752	.0025	.00
		125	4.6779	.0027	4.6779	.0027	.00
Sb	51	121	4.6810	.0046	4.6810	.0046	.00
		123	4.6890	.0051	4.6890	.0051	.00
Te	52	122	4.7100	.0053	4.7100	.0053	.00
		123	4.7120	.0045	4.7120	.0045	.00
		124	4.7190	.0046	4.7190	.0046	.00
		125	4.7210	.0046	4.7210	.0046	.00
		126	4.7282	.0038	4.7282	.0038	.00
		128	4.7350	.0046	4.7350	.0046	.00
		130	4.7420	.0046	4.7420	.0046	.00
I	53	127	4.7499	.0038	4.7499	.0039	-.01
Xe	54	124	4.7620	.0046	4.7620	.0046	.00
		126	4.7700	.0048	4.7700	.0048	.00
		128	4.7760	.0048	4.7760	.0048	.00
		129	4.7760	.0047	4.7760	.0047	.00
		130	4.7830	.0046	4.7830	.0046	.00
		131	4.7810	.0046	4.7810	.0046	.00
		132	4.7870	.0047	4.7870	.0047	.00
		134	4.7920	.0047	4.7920	.0047	.00

El	Z	A	R_{FOR} [fm]*	ΔR_{FOR}	R_{EXC} [fm]*	ΔR_{EXC}	$\frac{(R_{FOR}-R_{EXC})}{\Delta(R_{FOR}-R_{EXC})}$
Xe	(contd.)	136	4.7990	.0047	4.7990	.0047	.00
Cs	55	133	4.8040	.0046	4.8040	.0046	.00
Ba	56	134	4.8290	.0048	4.8290	.0048	.00
		135	4.8270	.0048	4.8270	.0048	.00
		136	4.8330	.0048	4.8330	.0048	.00
		137	4.8320	.0048	4.8320	.0048	.00
		138	4.8391	.0045	4.8391	.0046	.00
La	57	139	4.8550	.0049	4.8550	.0049	.00
Ce	58	140	4.8771	.0023	4.8771	.0025	.00
		142	4.9063	.0025	4.9063	.0026	.01
Pr	59	141	4.8920	.0050	4.8920	.0050	.00
Nd	60	142	4.9140	.0051	4.9140	.0051	.00
		143	4.9240	.0053	4.9240	.0053	.00
		144	4.9415	.0035	4.9415	.0035	.00
		145	4.9530	.0054	4.9530	.0054	.00
		146	4.9687	.0034	4.9687	.0035	.00
		148	4.9980	.0030	4.9980	.0031	.00
		150	5.0419	.0030	5.0419	.0031	-.01
Sm	62	144	4.9373	.0015	4.9373	.0015	.00
		147	4.9824	.0014	4.9824	.0014	.00
		148	5.0002	.0013	5.0002	.0013	.00
		149	5.0129	.0013	5.0129	.0013	.00
		150	5.0379	.0014	5.0379	.0014	.00
		152	5.0870	.0013	5.0870	.0013	.00
		154	5.1143	.0014	5.1143	.0014	.00
Eu	63	151	5.0440	.0053	5.0440	.0053	.00
		153	5.1180	.0055	5.1180	.0055	.00
Gd	64	154	5.1240	.0019	5.1240	.0019	.00
		155	5.1353	.0016	5.1353	.0016	.00
		156	5.1460	.0014	5.1460	.0014	.00
		157	5.1492	.0014	5.1492	.0014	.00
		158	5.1618	.0018	5.1618	.0018	.00
		160	5.1782	.0021	5.1782	.0021	.00
Tb	65	159	5.0600	.1500	5.0600	.1500	.00
Dy	66	161	5.1960	.0061	5.1960	.0061	.00
		162	5.2091	.0057	5.2091	.0058	.00
		163	5.2110	.0059	5.2110	.0059	.00
		164	5.2238	.0057	5.2238	.0058	.00
Ho	67	165	5.2022	.0308	5.2022	.0312	.00
Er	68	166	5.2390	.0060	5.2390	.0060	.00
		167	5.2580	.0060	5.2580	.0060	.00
		168	5.2713	.0060	5.2713	.0061	.00
		170	5.2847	.0061	5.2847	.0061	.00
Tm	69	169	5.2256	.0035	5.2256	.0035	.00
Yb	70	170	5.2860	.0063	5.2860	.0063	.00
		171	5.2534	.0295	5.2865	.0222	-.90

El	Z	A	R_{FOR} [fm]*	ΔR_{FOR}	R_{EXC} [fm]*	ΔR_{EXC}	$\frac{(R_{FOR}-R_{EXC})}{\Delta(R_{FOR}-R_{EXC})}$
Yb	(contd.)	172	5.2875	.0168	5.2981	.0151	-.47
		173	5.3060	.0063	5.3060	.0063	.00
		174	5.3033	.0258	5.3143	.0191	-.34
		176	5.3655	.0341	5.3320	.0251	.79
Lu	71	175	5.3700	.0300	5.3700	.0300	.00
Hf	72	176	5.3310	.0065	5.3310	.0065	.00
		177	5.3340	.0065	5.3340	.0065	.00
		178	5.3380	.0065	5.3380	.0065	.00
		179	5.3390	.0065	5.3390	.0065	.00
		180	5.3490	.0063	5.3490	.0063	.00
Ta	73	181	5.3531	.0063	5.3531	.0063	.00
W	74	182	5.3550	.0020	5.3550	.0020	.00
		183	5.3300	.1500	5.3300	.1500	.00
		184	5.3640	.0020	5.3640	.0020	.00
		186	5.3808	.0058	5.3808	.0062	.00
Os	76	186	5.3870	.0072	5.3870	.0072	.00
		188	5.4001	.0013	5.4001	.0013	.00
		189	5.3600	.1500	5.3600	.1500	.00
		190	5.4062	.0014	5.4062	.0014	.00
		192	5.4100	.0020	5.4100	.0020	.00
Ir	77	191	5.3900	.1500	5.3900	.1500	.00
		193	5.4100	.1500	5.4100	.1500	.00
Pt	78	194	5.3986	.0259	5.4043	.0195	-.17
		195	5.4270	.0069	5.4270	.0069	.00
		196	5.3937	.0160	5.3801	.0115	.69
		198	5.4410	.0063	5.4410	.0063	.00
Au	79	197	5.4379	.0046	5.4379	.0047	.00
Hg	80	198	5.4480	.0065	5.4480	.0065	.00
		199	5.4490	.0069	5.4490	.0069	.00
		200	5.4570	.0065	5.4570	.0065	.00
		202	5.4670	.0066	5.4670	.0066	.00
		204	5.4720	.0020	5.4720	.0020	.00
Tl	81	203	5.4674	.0041	5.4674	.0042	.00
		205	5.4752	.0038	5.4752	.0038	-.01
Pb	82	196	5.4420	.0105	5.4420	.0105	.00
		197	5.4420	.0105	5.4420	.0105	.00
		198	5.4500	.0085	5.4500	.0085	.00
		199	5.4500	.0085	5.4500	.0085	.00
		200	5.4590	.0075	5.4590	.0075	.00
		201	5.4610	.0075	5.4610	.0075	.00
		202	5.4690	.0055	5.4690	.0055	.00
		203	5.4710	.0055	5.4710	.0055	.00
		204	5.4793	.0013	5.4793	.0013	.00
		205	5.4820	.0035	5.4820	.0035	.00
		206	5.4896	.0012	5.4896	.0012	.00
		207	5.4938	.0013	5.4938	.0013	.00

El	Z	A	R_{FOR} [fm]*	ΔR_{FOR}	R_{EXC} [fm]*	ΔR_{EXC}	$\frac{(R_{FOR}-R_{EXC})}{\Delta(R_{FOR}-R_{EXC})}$
Pb	(contd.)	208	5.5010	.0012	5.5010	.0012	.02
		209	5.5110	.0025	5.5110	.0025	.00
		210	5.5230	.0035	5.5230	.0035	.00
		211	5.5330	.0055	5.5330	.0055	.00
		212	5.5450	.0075	5.5450	.0075	.00
		214	5.5650	.0105	5.5650	.0105	.00
Bi	83	209	5.5254	.0035	5.5253	.0038	.02
Th	90	232	5.7161	.0393	5.7463	.0291	-.62
U	92	233	5.8160	.0080	5.8160	.0080	.00
		234	5.8289	.0050	5.8289	.0050	.00
		235	5.8191	.0140	5.8262	.0118	-.39
		238	5.8473	.0098	5.8543	.0080	-.55
Pu	94	239	5.8300	.0200	5.8300	.0200	.00
Am	95	241	5.8929	.0035	5.8929	.0035	.00
		243	5.9047	.0035	5.9047	.0035	.00

[fm]* Radius data are given in fm units, an exception is the neutron where according to the conventions the fm^2 is shown

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NOTE by I. Angeli

Evaluation Procedures

A. The iterative "*rmsEVA*" procedure (FORTRAN). A detailed description of this procedure can be found in Ch. 4. of ref. 1. Here only the main steps will be given, and the same equation numbers are used as in that paper. The procedure begins with the calculation of the *weighted average* R_{av} of the input data $R_i \pm \Delta R_i$ (i=1, 2, …, n):

$$R_{av} = \frac{\sum_i w_i R_i}{\sum_i w_i} \qquad \text{where} \qquad w_i = \frac{1}{(\Delta R_i)^2} \qquad (1)$$

The *external error*

$$\Delta_{ext} R_{av} = \sqrt{\frac{\sum_i \frac{1}{\Delta R_i^2}(R_i - R_{av})^2}{(n-1)\sum_i \frac{1}{\Delta R_i^2}}} \qquad (8a)$$

measures the actual spread of the data R_i around R_{av}. On the other hand, the *internal error*

$$\Delta_{\text{int}} R_{av} = \sqrt{\frac{1}{\sum_i \frac{1}{\Delta R_i^2}}} \qquad (8b)$$

is a prediction on the uncertainty of the mean value. It is determined exclusively by the errors ΔR_i and does not depend on the spread of R_i values. Data groups may contain several data biased by undiscovered systematic errors. This results in a higher external error than predicted by the internal error. The present procedure gradually increases the input errors to make the *internal* and *external errors* equal. These new errors are formed by adding a common systematic error S to the original errors ΔR_i

$$\Delta R_i' = \Delta R_i + S \qquad (18a)$$

and now the new weights

$$w_i' = \frac{1}{(\Delta R_i')^2}$$

are used in Eq. (1) to calculate a new R_{av}'. The determination of the proper S values needs iteration. It should be remarked that the new grand mean R_{av}' formed with the w_i' weights generally differs from the first step value R_{av}. In our analysis, the weighted average value from the last step of the iteration (18a) was accepted as $R_{FOR}=(R_{av}')_{last}$ with an uncertainty $\Delta R_{FOR}=(\Delta_{int}R'_{av})_{last}=(\Delta_{ext}R'_{av})_{last}$.

B) The simplified procedure (*EXCEL)*. Here, the average radius and its uncertainty are calculated in a single step:

$$R_{EXC} = R_{av} = \frac{\sum_i R_i / \Delta R_i^2}{\sum_i 1/\Delta R_i^2} \qquad \Delta R_{EXC} = \Delta R_{av} = \max\left(\Delta_{\text{int}} R_{av},\ \sqrt{\tfrac{1}{2}\left((\Delta_{\text{int}} R_{av})^2 + (\Delta_{ext} R_{av})^2\right)}\right) \quad (19)$$